Mathematisches Institut
der Universität Bonn
P. Koepke, B. Irrgang

Winter 09/10
November 30, 2009
Exercise Sheet 7

Exercises for

Models of Set Theory II

Some notation: $\forall^\infty n$ means for all but finitely many $n \in \omega$, and $\exists^\infty n$ means for infinitely many $n \in \omega$.

25. Let $F \subseteq \omega^\omega$ of size κ be such that

$$\forall g \in \omega^\omega \exists f \in F \forall^\infty n \; f(n) \neq g(n).$$

Show that ω^ω is the union of κ meager sets.

26. Assume that $F \subseteq \omega^\omega$ is not meager. Show that

$$\forall g \in \omega^\omega \exists f \in F \exists^\infty n \; f(n) = g(n).$$

A set A of reals has strong measure 0 if for every sequence $a_0 \geq a_1 \geq \ldots \geq a_n \geq \ldots$ of positive reals there exists a sequence of open intervals I_n such that $\text{length}(I_n) \leq a_n$ and $A \subseteq \bigcup\{I_n \mid n \in \omega\}$.

27. Show that the Cantor set does not have strong measure 0.

28. Show that if $A \subseteq \mathbb{R}$ contains a perfect subset, then it does not have strong measure 0.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at December 7, 2009.