Mathematisches Institut
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Winter 09/10
November 9, 2009
Exercise Sheet 4

Exercises for

Models of Set Theory II

Let

$$\mathbb{H} = \{(s, E) \mid s \in \omega^{<\omega},\ E \subseteq \omega^{\omega} \text{ finite}\}.$$

For $(s, E), (t, F) \in \mathbb{H}$ set $(s, E) \leq (t, F)$ iff $t \subseteq s$, $F \subseteq E$ and $s(k) > f(k)$ for all $f \in F$ and all $k \in dom(s) - dom(t)$.

Let $f, g : \omega \to \omega$. We say that f eventually dominates g if $f(n) > g(n)$ for all but finitely many $n \in \omega$. A set $\mathfrak{G}$ of functions is eventually dominated by f if f eventually dominates every $g \in \mathfrak{G}$.

Assume that GCH holds. Consider the iteration $\langle \mathbb{P}_\alpha \mid \alpha \leq \omega_2 \rangle$ given by $\langle \dot{\mathbb{Q}}_\alpha \mid \alpha < \omega_2 \rangle$ where $\mathbb{P}_\alpha \Vdash \dot{\mathbb{Q}}_\alpha = \mathbb{H}$ for all $\alpha < \omega_2$.

13. Prove that $\mathbb{P}_{\omega_2}$ satisfies ccc.

14. Let G be $\mathbb{P}_{\omega_2}$-generic and $f \in V[G]$ be a function $f : \omega \to \omega$. Show that $f \in V[G_\alpha]$ for some $\alpha < \omega_2$.

Hint: Consider a nice name for f.

15. Show that $\mathbb{P}_{\omega_2} \Vdash 2^{\aleph_0} = \omega_2$.

16. Show $\mathbb{P}_{\omega_2} \Vdash$ (Every family $\mathfrak{G}$ of fewer that $2^{\aleph_0}$ functions from ω to ω is dominated by some $f : \omega \to \omega$).

Hint: Consider a nice name for a family $\mathfrak{G}$ of size $\leq \omega_1$ and prove a statement like in exercise 14.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at November 16, 2009.