Mathematisches Institut
der Universität Bonn
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Winter 09/10
January 11, 2010
Exercise Sheet 11

Exercises for

Models of Set Theory II

41. Show that there exists a sequence $\langle f_\alpha \mid \alpha \in \omega_1 \rangle$ such that:
(1) For every $\alpha < \omega_1$, $f_\alpha : \alpha \to \omega$ is injective.
(2) For all $\alpha < \beta < \omega_1$, $f_\alpha(\eta) = f_\beta(\eta)$ holds for all but finitely many $\eta < \alpha$.
(3) For all $\alpha < \omega_1$, $\omega \setminus rng(f_\alpha)$ is infinite.
Hint: Define the sequence recursively.

42.(continuation of exercise 41) Show that the set $\{f_\alpha \upharpoonright \beta \mid \alpha, \beta \in \omega_1\}$ ordered by inclusion is an Aronszajn tree T.

43. Let $\mathbb{P}$ be a countable forcing and G be $\mathbb{P}$-generic over V. Show that for every (in $V[G]$) uncountable $X \subseteq \omega_1$, $X \in V[G]$, there exists an (in V) uncountable set $Y \subseteq X$, $Y \in V$.

44.(continuation of exercise 42) For every $r : \omega \to \omega$ consider

$$T_r = \{r \circ (f_\alpha \upharpoonright \beta) \mid \alpha, \beta \in \omega_1\}.$$

Let G be Cohen-generic over V and $r := \bigcup G$. Show that T_r is a Suslin tree in $V[G]$.
Hint: Assume that $\{r \circ (f_{\alpha(\beta)} \upharpoonright \beta) \mid \beta \in A\}$ is an uncountable antichain in T_r. By exercise 43, there exists an uncountable $W \subseteq A$, $W \in V$ such that $\{r \circ (f_{\alpha(\beta)} \upharpoonright \beta) \mid \beta \in W\}$ is an uncountable antichain. Let $g_\beta := f_{\alpha(\beta)} \upharpoonright \beta$. Let p be a Cohen condition. Apply the Δ-system lemma to the sets $\{\xi < \beta \mid g_\beta(\xi) < dom(p)\}$ to show that for $\beta_1, \beta_2 \in W$ there exists $q \leq p$ with $q \Vdash (\check{r} \circ \check{g}_{\beta_1}$ and $\check{r} \circ \check{g}_{\beta_2}$ are compatible functions).

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at January 18, 2010.