Mathematisches Institut
der Universität Bonn
P. Koepke, B. Irrgang

Winter 09/10
December 21, 2009
Exercise Sheet 10

Exercises for

Models of Set Theory II

37. Show:
(a) $\mathfrak{d} = min\{card(F) \mid F \subseteq \omega^\omega,\ \forall g \in \omega^\omega \exists f \in F \forall n \in \omega\ g(n) \leq f(n)\}$
(b) $\mathfrak{b} \neq min\{card(F) \mid F \subseteq \omega^\omega,\ \forall g \in \omega^\omega \exists f \in F \forall n \in \omega\ f(n) \not\leq g(n)\}$.

Let $\mathcal{K}$ be the σ-complete ideal generated by the compact subsets of ω^ω.

38. Prove $add(\mathcal{K}) = non(\mathcal{K}) = \mathfrak{b}$.

39. Prove $cov(\mathcal{K}) = cof(\mathcal{K}) = \mathfrak{d}$.

40. Show that $\mathbb{R} \cap L$ is either null or not Lebesgue measurable if there exists a real which is not in L (the constructible universe).
Hint: Consider $S = [0,1] \cap L$ and $S_n = \{x + \frac{a}{n} \mid x \in S\}$ where $a \in \mathbb{R} - L$.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at January 11, 2010.

Merry Christmas and a Happy New Year