Mathematisches Institut
der Universität Bonn
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Winter 09/10
October 14, 2009
Exercise Sheet 1

Exercises for

Models of Set Theory II

1. Let M be a ground model where $2^\omega = 2^{\omega_1} = \omega_2$ holds. Let

$$\mathbb{P} = \{p : n \to \omega_1 \mid n \in \omega\}^M$$

and for $p, q \in \mathbb{P}$ set $p \leq q$ iff $p \supseteq q$. Let G be M-generic for $\mathbb{P}$. Show that CH holds in $M[G]$.

2. Let ϕ be the statement that whenever $\mathbb{P}$ is an ω_1-closed ω_2-cc forcing and $\mathcal{D}$ is a family of $< 2^{\omega_1}$ dense sets in $\mathbb{P}$ there is a $\mathcal{D}$-generic filter on $\mathbb{P}$. Show that $\neg CH \ \vee \ 2^{\omega_1} = \omega_2$ implies ϕ.

3. Let $\mathbb{P} = \{p : \eta \to \omega_2 \mid \eta \in \omega_1\}$. Show that there exists a set $\mathcal{D}$ of ω_2 in $\mathbb{P}$ dense sets for which there is no $\mathcal{D}$-generic filter.

4. Assume MA_κ. Let $\mathcal{F}$ be a family of κ Lebesgue measurable subsets of $\mathbb{R}$. Show that $\bigcup \mathcal{F}$ is Lebesgue measurable and $\lambda(\bigcup \mathcal{F}) = \lambda(\bigcup \mathcal{F}')$ for some countable $\mathcal{F}' \subseteq \mathcal{F}$.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at October 21, 2009.