Mathematisches Institut
der Universität Bonn
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Summer Semester 2009
June 22, 2009
Exercise Sheet 8

Exercises for

Models of Set Theory I

Let MA be the following statement: If $\mathbb{P}$ is a forcing which satisfies the ccc and if $card(\mathfrak{D}) < 2^{\aleph_0}$, then there exists a filter G on $\mathbb{P}$ such that $G \cap D \neq \emptyset$ for all $D \in \mathfrak{D}$ which are dense in $\mathbb{P}$.

29. Prove that CH implies MA.

30. Show that the above statement cannot hold if we replace $card(\mathfrak{D}) < 2^{\aleph_0}$ by $card(\mathfrak{D}) = 2^{\aleph_0}$.

31. Assume MA and $\aleph_1 < 2^{\aleph_0}$. Show that then there exist no Suslin trees.
Hint: If $(T, \leq)$ is a (normal) Suslin tree, then $\mathbb{P} = (T, \geq)$ is a ccc forcing that adds a branch through T.

Let $\mathfrak{G}$ be a set of functions $f : \omega \to \omega$. Let

$$\mathbb{H} = \{(s, E) \mid s \in \omega^{<\omega},\ E \subseteq \mathfrak{G} \text{ finite}\}.$$

For $(s, E), (t, F) \in \mathbb{H}$ set $(s, E) \leq (t, F)$ iff $t \subseteq s$, $F \subseteq E$ and $s(k) > f(k)$ for all $f \in F$ and all $k \in dom(s) - dom(t)$.
Let $f, g : \omega \to \omega$. We say that f eventually dominates g if $f(n) > g(n)$ for all but finitely many $n \in \omega$. A set $\mathfrak{G}$ of functions is eventually dominated by f if f eventually dominates every $g \in \mathfrak{G}$.

32. Use $\mathbb{H}$ to prove: MA implies that every family $\mathfrak{G}$ of fewer that $2^{\aleph_0}$ functions from ω to ω is dominated by some $f : \omega \to \omega$.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at June 29, 2009.