Exercises for

Models of Set Theory I

13. Let $F : Ord \to V$ be a class which is definable without parameters, i.e. $F = \{x \mid \varphi(x)\}$ for some $\in$-formula φ. Prove that $rng(F) \subseteq OD$. Thus OD is the largest class for which there exists a bijection $F : Ord \to OD$ definable without parameters.

14. Prove that HOD is the largest transitive model of ZF for which there exists a bijection $F : Ord \to HOD$ which is definable without parameters.

15. Prove for every L_α:
(a) If $x, y \in L_\alpha$, then $x \cup y, x \cap y, x - y, \bigcup x \in L_\alpha$.
(b) If $x \in L_\alpha$, then $TC(x) \in L_\alpha$.

16. Prove:
(a) If x is a set such that $x \subseteq L$, then there exists an $\alpha \in Ord$ such that $x \subseteq L_\alpha$.
(b) If $V \neq L$, then there exists a set x such that $x \notin L$ but $x \subseteq L$.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at May 20, 2009.