Colloquium Logicum 2006
September 22-24, 2006, Bonn, Germany

English follows German

Tagung der Deutschen Vereinigung für Mathematische Logik und für Grundlagen der Exakten Wissenschaften (DVMLG)

Das Colloquium Logicum 2006 (CL2006) findet im Anschluss an die Jahrestagung 2006 der Deutschen Mathematiker Vereinigung (DMV) in Bonn statt. Das Colloquium Logicum beginnt am Freitag, dem 22. September um 14:15 Uhr und endet am Sonntag, dem 24. September um 16:00 Uhr.

Das wissenschaftliche Programm besteht aus

- einem Gödel-Vortrag und sechs weiteren Hauptvorträgen zu mathematischer Logik, Komplexitätstheorie, Geschichte der Logik und Philosophie der Mathematik
- vier Plenarvorträgen, in denen hervorragende Dissertationen vorgestellt werden
- zwei (zweiten Hälften von) Minisymposien, die gemeinsam mit der DMV durchgeführt werden: "The use of proof theory in mathematics", organisiert von P. Schuster (München) und "Set theory", organisiert von E. Schimmerling (Pittsburgh) und R. Schindler (Münster)
- und sieben eingereichten Vorträgen aus der Logik und verwandten Gebieten.

Die Mitgliederversammlung der DVMLG findet während des Colloquium Logicums am Samstagabend statt.

Die DVMLG dankt: dem Mathematischen Institut der Universität Bonn für die Bereitstellung von Räumen und Geräten, der Deutschen Forschungsgemeinschaft (GZ: 4853/58/06) für eine Zuwendung zur Finanzierung eingeladener Sprecher und der DMV-Tagung mit ihrem Vorsitzenden Prof. Ballmann für die Möglichkeit, einige Strukturen der Jahrestagung mitbenutzen zu können.

Detaillierte Informationen zum Colloquium Logicum 2006 finden sich hier und auf dieser Webseite:
`http://www.math.uni-bonn.de/people/dvmlg/cl2006/`

Biennial meeting of the German Society for Mathematical Logic (DVMLG)

Colloquium Logicum 2006 (CL 2006) will take place right after the annual meeting of the German Mathematical Society (DVM) at Bonn. CL 2006 will start on Friday, September 22 at 2:15 pm and finish on Sunday, September 24 at 4 pm.

The scientific program consists of

- a Gödel lecture and six further 1-hour plenary talks on mathematical logic, complexity theory, history of logic, and on the philosophy of mathematics
- four plenary talks presenting excellent doctoral dissertations
- two (second halves of) minisymposia which are held in conjunction with the DMV-meeting: "The use of proof theory in mathematics", organized by P. Schuster (München) and "Set theory", organized by E. Schimmerling (Pittsburgh) and R. Schindler (Münster)
- and seven contributed talks on logic and related areas.

The general assembly of the DVMLG will take place on Saturday evening after the talks.

The DVMLG expresses thanks to the Mathematical Institute of the University of Bonn for rooms and equipment, to the Deutsche Forschungsgemeinschaft for a grant for invited speakers, and to the DMV meeting with its chairman Prof. Ballmann for allowing the use of some structures of the annual meeting.

For detailed information on the Colloquium Logicum 2006, look at this program and at this web page:
`http://www.math.uni-bonn.de/people/dvmlg/cl2006/`

Schedule

kl.Hs = kleiner Hörsaal / Wegelerstraße 10
Zs. = Zeichensaal / Wegelerstraße 10

FRIDAY, SEPTEMBER 22

14:30-14:45	kl.Hs.	Opening
		Minisymposium: the use of proof theory in mathematics
15:00-15:50	Zs.	Monika Seisenberger (Swansea) Program Extraction from Proofs: Theory and Practice
16:00-16:50	Zs.	Helmut Schwichtenberg (München) Logic for Computable Functionals and their Approximations
17:00-17:50	Zs.	Thomas Streicher (Darmstadt) Shoenfield = Gödel after Krivine
		Minisymposium: set theory
15:00-15:20	kl.Hs.	Gunter Fuchs (Münster) Degrees of Rigidity for Souslin Trees and Changing the Heights of Automorphism Towers
15:30-15:50	kl.Hs.	David Asperó (Barcelona) Definable well–orders of $H(\omega_2)$ and forcing axioms
16:00-16:20	kl.Hs.	Natasha Dobrinen (Wien) Co-stationarity of the ground model
16:30-16:50	kl.Hs.	Otmar Spinas (Kiel) Perfect Set Theorems
17:00-17:50	kl.Hs.	Jouko Väänänen (Helsinki) Strong Logics

SATURDAY, SEPTEMBER 23

9:00-10:00	kl.Hs.	Steffen Lempp (Madison) On bounds on the computability-theoretic complexity of trivial, strongly minimal models, and bounds on the complexity of axiomatizations of trivial, strongly minimal theories
10:00-10:30		*Coffee break*

10:30-11:30	kl.Hs.	Carlo Cellucci (Rome) The Generalization Problem: What Entitles Us to Pass from Particular Premisses to General Conclusions?
11:30-12:30	kl.Hs.	Thomas Schwentick (Dortmund) Two variable logics in the presence of an equivalence relation
12:30-14:00		*Lunch break*
14:00-14:45	kl.Hs.	Katherine Thompson (Vienna) Methods for solving universality problems (PhD)
14:45-15:30	kl.Hs.	Andrea Reichenberger (Paderborn) Hilbert's Axiom of Solvability Revisited in the Context of the Ignorabimus-Dispute (PhD)
15:30-16:00		*Coffee break*
16:00-17:00	kl.Hs.	Stan Wainer (Leeds) Proof Theory of "Predicative" Arithmetic
17:00-17:50		Contributed talks:
17:00-17:25	kl.Hs.	Peter Schuster (München) Acceptable Poset Properties for the Hilbert Basis Theorem
17:00-17:25	Zs.	Philip Welch (Bristol) The Inner Model Hypothesis
17:25-17:50	kl.Hs.	Petr Hájek (Prague) Mathematical fuzzy logic - fuzzy logic taken seriously
17:50-18:15		*Break*
18:15	kl.Hs.	DVMLG meeting

SUNDAY, SEPTEMBER 24

9:00-9:45	kl.Hs.	Dieter Probst (Bern) How to apply pseudo-hierarchy arguments outside second order arithmetic (PhD)
9:45-10:30	kl.Hs.	Dietmar Berwanger (Bordeaux) The discerning power of games (PhD)
10:30-11:00		*Coffee break*

11:00-12:00	kl.Hs.	Wilfried Sieg (Pittsburgh) Gödel on computability: “The human mind infinitely surpasses?” (Gödel Lecture)
12:00-13:00		*Sandwiches*
13:00-13:50		Contributed talks:
13:00-13:25	kl.Hs	Brian Semmes (Amsterdam) On the structre of Baire Class 2 functions
13:00-13:25	Zs.	Serikzhan Badaev (Almaty) On uniform non-monotone enumerations
13:25-13:50	kl.Hs.	Karsten Steffens (Hannover) On the foundations of pcf-theory
13:25-13:50	Zs.	Sujata Ghosh (Amsterdam) A bi-logic for belief-disbelief
14:00-15:00	kl.Hs.	Martin Hils (Berlin) From strongly minimal fusion to the construction of a bad field
15:00-16:00	kl.Hs.	Ralf Schindler (Münster) If every uncountable cardinal is singular, then AD holds in $\mathsf{L}(\mathbf{R})$

Abstracts

Invited talks

DAVID ASPERÓ (Barcelona)
Definable well–orders of $H(\omega_2)$ and forcing axioms
Fri 22.9.2006, 15:30-16:00, kl.Hs.

This talk deals with the problem of building set-forcing extensions in which there is a simple definition, over the structure $\langle H(\omega_2), \in\rangle$ and without parameters, of a prescribed member of $H(\omega_2)$ or of a well–order of $H(\omega_2)$, possibly together with some strong forcing axiom.

I will present two theorems. The first one is an optimal result, with respect to the logical complexity of the definitions involved, at the level of the structure $\langle H(\omega_2), \in, NS_{\omega_1}\rangle$. This result is a particular case of a much more general theorem applying to $H(\kappa^+)$ for every uncountable regular cardinal κ.

The second theorem I will present says that, under the assumption that there is a supercompact cardinal, there is a partial order forcing both the existence of a well–order of $H(\omega_2)$ definable, over $\langle H(\omega_2), \in\rangle$, by a formula without parameters, and that the forcing axiom PFA^{++} holds.

DIETMAR BERWANGER (Bordeaux)
The discerning power of games
Sun 24.9.2006, 9:45-10:30, kl.Hs.

Game-centered methods are fundamental in the study of descriptive and computational complexity of logical formalisms. We present a toolbox of games for the μ-calculus, and show how it can be used to develop a pertinent measure of graph complexity and to prove a separation theorem for the variable hierarchy of this logic.

CARLO CELLUCCI (Rome)
The Generalization Problem: What Entitles Us to Pass from Particular Premisses to General Conclusions?
Sat 23.9.2006, 10:00-11:00, kl.Hs.

It is generally assumed that Gentzen's justification of universal generalization is unproblematic. In fact Gentzen provides two such justifications, which however turn out to be both inadequate. In my talk I will consider an alternative justification of universal generalization, which yields a new formulation of that rule. Such justification depends on a certain view of mathematical objects and proofs. I will also show that there is a connection between universal generalization and a non-deductive rule, the analogy rule.

NATASHA DOBRINEN (Vienna)
Co-stationarity of the ground model
Fri 22.9.2006, 16:00-16:30, kl.Hs.

The bulk of this talk is based on joint work with Sy-David Friedman. Given $V \subseteq W$ models of ZFC with the same ordinals and $\kappa < \lambda$ cardinals in W with κ regular, let $\mathcal{P}_\kappa(\lambda)$ denote the collection of subsets of λ of size less than κ in W. We say that the ground model is *co-stationary* if $\mathcal{P}_\kappa(\lambda) \setminus V$ is stationary in $\mathcal{P}_\kappa(\lambda)$. Gitik showed the following: Suppose κ is a regular cardinal in W, and λ is greater than or equal to $(\kappa^+)^W$. If there is a real in $W \setminus V$, then the ground model is co-stationary in $\mathcal{P}_\kappa(\lambda)$.

We consider problems of generalizing Gitik's Theorem to forcing extensions in which no reals are added. In particular, we show that the analogue of Gitik's Theorem for $\aleph_2$-c.c. forcings which add a new subset of $\aleph_1$ (but no new ω-sequences) is equiconsistent with a class of Erdös cardinals. The necessity of ω_1-Erdös cardinals follows from a covering theorem of Magidor. For regular $\kappa \geq \aleph_2$ with $\aleph_\kappa > \kappa$, the co-stationarity of the ground model in the $\mathcal{P}_{\kappa^+}(\aleph_\kappa)$ of a κ-Cohen forcing extension is equiconsistent with κ measureable cardinals.

For $\nu \geq \aleph_1$ we present some consistency results concerning partial orderings which add a new ν-sequence but no new subset of ν. We also include some more recent work with Justin Moore concerning partial orderings which add a new ω-sequence without adding a new real.

GUNTER FUCHS (Münster)

Degrees of Rigidity for Souslin Trees and Changing the Heights of Automorphism Towers

Fri 22.9.2006, 15:00-15:30, kl.Hs.

Various strong notions of rigidity for Souslin trees are investigated and separated, assuming the diamond principle, into a hierarchy. Most of these rigidity properties state that a tree has a certain rigidity property in any model obtained by forcing with the tree itself.

An application to the automorphism tower problem is given, showing that, again assuming diamond, there is a group the height of whose automorphism tower is highly malleable by forcing with certain Souslin trees. Carrying out the construction at higher cardinality levels gives the full statement on changing the heights of automorphism towers, that was realized by Hamkins and Thomas using proper class forcing, in L.

MARTIN HILS (Berlin)

From strongly minimal fusion to the construction of a bad field

Sun 24.9.2006, 14:00-15:00, kl.Hs.

Modifying Fraissé's amalgamation method, Ehud Hrushovski constructed a strongly minimal set with a non-locally modular geometry which does not come from an algebraically closed field. He thus refuted the Zilber trichotomy. Using the same technique, he managed to "fuse" two strongly minimal sets into a single one. Hrushovski's amalgamation technique has been extended to constructions over vector spaces. This lead to the construction of a new uncountably categorical group (nilpotent of class 2) and more recently to the construction of various algebraically closed fields with extra structure, in particular a field of finite Morley rank with a distinguished proper infinite additive subgroup (in positive characteristic) and a field of finite Morley rank with a distinguished proper infinite multiplicative subgroup (in characteristic 0), i.e. a *bad field*. The latter is joint work with A. Baudisch, A. Martin-Pizarro and F. Wagner.

We give the main ideas of the method, with particular emphasis on the construction of a bad field.

STEFFEN LEMPP (Madison)

On bounds on the computability-theoretic complexity of trivial, strongly minimal models, and bounds on the complexity of axiomatizations of trivial, strongly minimal theories

Sat 23.9.2006, 9:00-10:00, kl.Hs.

In joint work with Goncharov, Harizanov, Laskowski, and McCoy, we showed [1] that any trivial, strongly minimal (and thus uncountably categorical) theory is model complete after naming constants. A direct consequence is that each such theory T is $\exists\forall\exists$-axiomatizable, and that if one model of T is computable then all countable models are $0''$-decidable. (This result was subsequently extended by Dolich, Laskowski and Raichev [2] to trivial theories of Morley rank 1 but arbitrary Morley degree.)

We present a joint result with Khoussainov and Solomon [3] that this result is optimal by showing that there is a trivial, strongly minimal computable prime model such that all other countable models of its theory code $0''$ (i.e., $0''$ is computable from the open diagrams of these models). We also present some subsequent work of Laskowski from the same paper on axiomatizations.

Finally, we discuss some on-going research with Dolich and Laskowski on extending the original result to Morley rank 2 and beyond, when only weakenings of model completeness are possible, trying to further explore this connection between axiomatizability and computability.

[1] Goncharov, Sergey S.; Harizanov, Valentina S.; Laskowski, Michael C.; Lempp, Steffen; and McCoy, Charles F. D.: Trivial, strongly minimal theories are model complete after naming constants, Proc. Amer. Math. Soc. 131 (2003), 3901-3912.

[2] Dolich, Alfred; Laskowski, Michael C.; and Raichev, Alexander: Model completeness for trivial, uncountably categorical theories of Morley rank one, with A. Dolich and A. Raichev, Archive for Math. Logic (to appear).

[3] Khoussainov, Bakhadyr M.; Laskowski, Michael C. Laskowski, and Solomon, D. Reed: On the computability-theoretic complexity of trivial,

strongly minimal models, to appear.

DIETER PROBST (Bern)

How to apply pseudo-hierarchy arguments outside second order arithmetic

Sun 24.9.2006, 9:00-9:45, kl.Hs.

Especially in second order arithmetic, pseudo-hierarchy arguments have become a powerful tool. Typical applications are the existence proofs for specific fixed point definitions (cf. Avigad [1]) or the pairwise equivalence of (ATR), the Perfect Set Theorem and Σ^0_1 determinacy (cf. Simpson [3]). We show, how to apply pseudo-hierarchy arguments for instance in admissible set theory without foundation.

Basically, a *hierarchy* h for an operator F is a function whose domain is a linear ordering $\prec$, and $h(\alpha) = F(\dot{\bigcup}_{\beta \prec \alpha} h(\beta))$ holds for each $\alpha \in \mathsf{Dom}(h)$. If $\prec$ is not well-founded, then h is called a *pseudo-hierarchy*. Whereas in many theories, a recursion theorem allows to iterate a suitable class of operators along well-orderings, the existence of pseudo-hierarchies has to be shown by different means: In second order arithmetic, their existence is due to the fact that being a well-ordering is not expressible by a Σ formula. However, the corresponding statement does not hold in standard models of admissible set theory. Alternatively, we show that the proof-theoretic strength of a wide range of theories remains unaffected when adding an axiom that asserts the existence of pseudo-hierarchies.

Having pseudo-hierarchies at hand enables us to adapt many important results from second order arithmetic to admissible set theory without foundation, such as the equivalence of certain fixed point and iteration principles, the connection between reflection principles and choice schemas and characterizations of admissible classes in terms of the constructible hierarchy. Moreover, we gain new insights on fixed points of positive inductive definitions in second order arithmetic itself, which leads us to the answer of an old question asked by Feferman in his paper on Hancock's conjecture [2] about the strength of ID^*_1, the theory obtained by restricting fixed point induction of the well-known theory ID_1 to formulas that contain fixed point constants only positively.

[1] Jeremy Avigad, *On the relationship between ATR_0 and $\widehat{ID}_{<\omega}$*, The Journal of Symbolic Logic **61** (1996), no. 3, 768–779.

[2] Solomon Feferman, *Iterated inductive fixed-point theories: application to Hancock's conjecture*, The Patras Symposion (G. Metakides, ed.), North Holland, Amsterdam, 1982, pp. 171–196.

[3] Stephen G. Simpson, *Subsystems of Second Order Arithmetic*, Perspectives in Mathematical Logic, Springer-Verlag, 1998.

ANDREA REICHENBERGER (Paderborn)

Hilbert's Axiom of Solvability Revisited in the Context of the Ignorabimus-Dispute

Sat 23.9.2006, 14:45-15:30, kl.Hs.

In his famous address "Mathematical Problems" (Paris, 1900) Hilbert presented a list of unsolved problems, among them Cantor's Continuum Hypothesis (first problem), the problem to proof the consistency of the arithmetical axioms (second problem) and the problem to find an algorithm for determining of any given Diophantine equation whether or not it has any integer solutions (tenth problem). Hilbert was not only confident about the solvability of these problems. Even more, Hilbert believed "that every definite mathematical problem must necessarily be susceptible of an exact settlement, either in the form of an actual answer to the question asked, or by a proof of the impossibility of its solution and therewith the necessary failure of all attempts." (Hilbert 1900: 297) My talk deals with the question of how to interpret Hilbert's "axiom of solvability", i.e. the conviction of the solvability of every mathematical problem. I will try to clarify its relationship to Hilbert's "no ignorabimus"-thesis and to the decision problem by distinguishing between general solvability and formal decidability.

RALF SCHINDLER (Münster)

If every uncountable cardinal is singular, then AD holds in $\mathsf{L}(\mathbf{R})$

Sun 24.9.2006, 15:00-16:00, kl.Hs.

Moti Gitik has shown that "ZF + every uncountable cardinal is singular" is consistent (relative to strongly compact cardinals). We develop the method of the core model induction in a $\neg$AC context, and we show that if every uncountable cardinal is singular, then AD holds in $\mathsf{L}(\mathbf{R})$ of a generic extension of HOD. This is joint work with Daniel Busche.

THOMAS SCHWENTICK (Dortmund)

Two variable logics in the presence of an equivalence relation

Sat 23.9.2006, 11:30-12:30, kl.Hs.

Mortimer showed more than 30 years ago that two-variable logics possesses the Finite Model Property and therefore has a decidable satisfiability problem. Since then, this result has been refined and extended in several ways. In particular, satisfiability on structures with specific properties has been investigated. Of course, the general decidability result can be directly transfered only if the property at hand can be axiomatized with two variables.

Recently, the satisfiability problem for two-variable logics on structures with (one or more) equivalence relations has been investigated in more detail. Partially, these investigations were motivated by the fact that adding an equivalence relation to a structure has the same effect as associating a data value to each element and allowing for equality tests on these data values.

After a survey of some general results, the focus of the talk will be on recent results for (finite and infinite) strings and trees with one equivalence relation.

HELMUT SCHWICHTENBERG (München)
Logic for Computable Functionals and their Approximations
Fri 22.9.2006, 16:00-17:00, Zs.

An attempt is made to develop a constructive theory of formal neighborhoods for continuous functionals, in a direct and intuitive style. Guided by abstract domain theory, we consider a more concrete and (in the case of finitary free algebras) finitary theory of representations. As a framework for this we use Scott's information systems.

MONIKA SEISENBERGER (Swansea)
Program Extraction from Proofs: Theory and Practice
Fri 22.9.2006, 15:00-16:00, Zs.

This talk will give an overview of various techniques for extracting computational content from formal proofs emphasising the gap between pure methods that work in principle and refined techniques that can be applied to nontrivial examples with practically useful results.

WILFRIED SIEG (Pittsburgh)
Gödel on computability: "The human mind infinitely surpasses ?"
Sun 24.9.2006, 11:00-12:00, kl.Hs.

Gödel's role in the emergence of computability theory is, as John Dawson noted in a recent lecture, "dichotomous". There are crucial impulses, for example, the definition of "general recursive functions" in his 1934 Princeton Lectures that is the starting point for Kleene's work in recursion theory and that served as the rigorous mathematical notion in Church's first published formulation of his thesis in 1935. There is, however, no systematic body of recursion theoretic work or even an isolated central theorem associated with Gödel's name. The reason for that is clear, it seems to me: Gödel was not interested in the development of the theory per se, but strove for a secure conceptual foundation. He needed that foundation for two central and related purposes, namely, (i) to formulate the incompleteness theorems in full mathematical generality for all formal theories, and (ii) to articulate and sharpen philosophical consequences of the undecidability and incompleteness results.

It is this general foundational context that frames my discussion. The latter comes in five parts: I. The incompleteness theorems; II. General recursion; III. Finite machines; IV. Mechanical computors; V. Intelligent machinery. Under I, I discuss the intellectual setting of Gödel's announcement of the very early formulation of his first incompleteness theorem in Königsberg and the subsequent exchange with von Neumann and Herbrand. Then, under II, I introduce the equational calculus, Gödel's definition of general recursive functions and its absoluteness; that is followed under III by Gödel's remarks about mechanical procedures and Turing machines. Under IV, I give a sharpened and condensed version of Turing's analysis leading to an axiomatic formulation of computability that Gödel had intimated in 1934, but never followed up. That finally allows me to address the question, "Does the human mind infinitely surpass any finite machine?" - To hear answers to the question, well, you have to come to my lecture.

References to the works mentioned can be found in my paper, Gödel on computability, Philosophia Mathematica 14, 2006, 189-207.

OTMAR SPINAS (Kiel)
Perfect Set Theorems
Fri 22.9.2006, 16:30-17:00, kl.Hs.

I will present several results and open problems in the context of searching for perfect set theorems for the following largeness conditions for subsets of Cantor or Baire space: splitting property, refining property, infinitely often equal property.

THOMAS STREICHER (Darmstadt)
Shoenfield = Gödel after Krivine
Fri 22.9.2006, 17:00-18:00, Zs.

In the 1960s J. Shoenfield came up with a functional interpretation $(-)^S$ of Peano arithmetic (PA). Recently, G. Mints raised the question whether one can express $(-)^S$ as $(A^K)^D$ where D is Gödel's Dialectica interpretation and $(-)^K$ is an appropriately chosen negative translation.

We present such a translation $(-)^K$ going back to J.-L. Krivine and elaborated by B. Reus and T. Streicher, and prove that if

$$A^S \equiv \forall u \exists x A_S(u,x) \quad \text{and} \quad (A^K)^D \equiv \exists f \forall u A_D^K(f,u)\,,$$

then $A_D^K(f,u)$ and $A_S(u,f(u))$ are provably equivalent in HA_ω.

The content of this talk is joint work with Ulrich Kohlenbach.

KATHERINE THOMPSON (Vienna)

Methods for solving universality problems

Sat 23.9.2006, 14:00-14:45, kl.Hs.

We will discuss a number of ways of showing that universal models do or do not exist. The methods stem from model theory, set theory and category theory. We will see examples of these methods mostly using relational structures, but they can be applied to algebraic and topological structures as well. By comparing which methods work for different structures, one can find patterns in the behaviour of these structures with regard to universality.

JOUKO VÄÄNÄNEN (Helsinki)

Strong Logics

Fri 22.9.2006, 17:00-18:00, kl.Hs.

I will describe strong logics that arise naturally in database theory. I will discuss set theoretic questions related to their model theory in the infinite context. In particular, I will talk about recent joint work with Magidor on Löwenheim-Skolem type properties of strong logics.

STAN S. WAINER (Leeds)

Proof Theory of "Predicative" Arithmetic

Sat 23.9.2006, 16:00-17:00, kl.Hs.

This talk will survey recent work on a new hierarchy of proof systems, based on a weak form of induction formalising the underlying principles of Nelson's Predicative Arithmetic, and related to previous work of Leivant and of Girard. The hierarchy has a polynomial-time system at its base, and refines the classical systems PA, ID_1, ID_2, etc. in such a way that the natural bounding functions are now "slow growing" rather than "fast growing". Thus, for example, since the slow growing functions below the Bachmann-Howard ordinal are only as strong as the fast growing ones below ϵ_0, the hierarchy captures full PA by adding to the basic predicative system (which only has elementary recursive strength) the axioms for one inductive definition.

Contributed talks

SERIKZHAN BADAEV (Almaty)

On uniform non-monotone enumerations

Sun 24.9.2006, 13:00-13:25, Zs.

Informally, computable numbering of a family of c.e. sets is a surjective mapping of the set of natural numbers onto the family equipped with an algorithmic uniform enumeration procedure for its sets. The growing of the sets while the enumeration procedure proceeded is obviously monotone in the processing time. This means that every number (treated as a piece of information) enumerated in any set via uniform enumeration procedure never leaves this set in future. Thus the information accumulated in every set of the family is growing more and more.

Straightforward generalization leads anybody to the notion of computable numberings for the families of sets in arithmetical and Ershov's hierarchies. In these cases, any number could enter a set and it could leave the set later, and again enter it and etc during some algorithmic uniform enumeration procedure. In the case of the hierarchy of Ershov, number of these 'enter-leaves' is bounded by the level of hierarchy in which considered family is involved while in the case of the arithmetical one, it is usually required for a number of 'enter-leaves' to be finite only. In the latter case, one can use oracle in the computations and they becomes monotone modulo oracle. In contrary, in the case of Ershov's hierarchy, oracles could not be used and we have a pure non-monotone computations.

Our talk is devoted to show considerable differences between outputs of monotone and non-monotone uniformly computable procedures. This differences will be discussed in terms of Rogers semilattices of computable numberings.

SUJATA GHOSH (Amsterdam)

A bi-logic for belief-disbelief

Sun 24.9.2006, 13:25-13:50, Zs.

A logic for belief-disbelief change accommodating contraction and replacement has been defined, a semantics is given and soundness-completeness theorem is proved. The logic generalizes some of the earlier models.

PETR HÁJEK (Prague)

Mathematical fuzzy logic – fuzzy logic taken seriously

Sat 23.9.2006, 17:25-17:50, kl.Hs.

The term "fuzzy logic" is used in broad sense as synonymous with "theory and applications of fuzzy sets; but in the narrow sense it means indeed a logic that has undergone, in last decade, an intensive development as a kind if many-valued logic with a comparative notion of truth (sentences may be more or less true). This contribution wants to be a very brief survey presenting also some fresh still unpublished results.

The standard set of truth values is the real unit interval $[0, 1]$; starting connectives are conjunction $\&$ and implication $\rightarrow$, $\bar{0}$ is the truth constant for 0 (falsity). In the basic fuzzy logic BL, any continuous t-norm can be used as the truth function of conjunction and its residuum as the truth function of implication (definitions will be presented). Negation $\neg$ and the lattice connectives $\vee, \wedge$ are defined connectives. The general semantics uses so-called BL-algebras as algebras of truth functions of our connectives; there is a simple axiom system complete with respect to both semantics. Fixing a particular t-norm one gets stronger logics, notably Łukasiewicz, Gödel and product logic. Computational complexity of these logics is known.

Corresponding predicate logics have also double semantics – given a crisp non-empty domain, predicates are interpreted as fuzzy relations (of corresponding arity), with values in $[0, 1]$ (standard case) or in a (linearly ordered) BL-algebra (general semantics). Whereas the general semantics has a complete recursive axiomatization, for standard semantics the arithmetical complexity of the set of tautologies varies from Σ_1-complete (Gödel logic) over Π_2-complete (Łukasiewicz) to non-arithmetical (product logic, BL). Recently fragments of our logics (both propositional and predicate) resulting by deleting some connectives have been studied and classified. Also it was shown that the set of tautologies of the $\&$-free fragment of the product predicate logic is (still) non-arithmetical.

This kind of logic may be well called *mathematical fuzzy logic* (fuzzy logic as a branch of mathematical logic). It is a developing domain with a nice international cooperation. See http://plato.stanford.edu/entries/logic-fuzzy/ for a survey with references.

PETER SCHUSTER (JOINT WORK WITH HERVÉ PERDRY) (München)

Acceptable Poset Properties for the Hilbert Basis Theorem

Sat 23.9.2006, 17:00-17:25, kl.Hs.

We present a synthetical approach to a constructive theory of Noetherian rings that is suited for several competing versions of this notion.

BRIAN SEMMES (Amsterdam)
On the structure of Baire Class 2 functions
Sun 24.9.2006, 13:00-13:25, kl.Hs.

In this talk I will review the Tree game and show how it can be modified to characterize the Baire Class 2 functions. Time permitting, I will discuss recent efforts to obtain structure theorems by adapting the Jayne-Rogers method to the game theoretic setting.

KARSTEN STEFFENS (Hannover)
On the foundations of pcf-theory
Sun 24.9.2006, 13:25-13:50, kl.Hs.

Denote by Ideal(A) the set of all $\mathbf{proper\ ideals}$ on A. Let $I \in$ Ideal(A), let $I^+ = \{B \subseteq A : B \notin I\}$ be the set of $\mathbf{positive\ sets}$ and, for $B \in I^+$, call $I \restriction B = \{C \subseteq A : C \cap B \in I\}$ the $\mathbf{restriction\ of\ I}$ on B. If $A \setminus B \notin I$, then $I[B] = \langle I \cup \{B\}\rangle$ denotes the $\mathbf{ideal\ generated\ by}$ $I \cup \{B\}$. A function τ : Ideal $(A) \longrightarrow$ CN is called $\mathbf{continuous}$ if $\tau(\bigcap \Gamma) = \bigcap \tau[\Gamma]$ for each nonempty set $\Gamma \subseteq$ Ideal(A). I is said to be $\lambda\mathbf{-stable}$ if $\tau(J) = \lambda$ for all $J \in$ Ideal(A) such that $I \subseteq J$. Let $(H, <)$ be a partially ordered set. The cardinal number $\min\{|X| : X \subseteq H, \neg\exists h \in H\ X < h\}$ is said to be the $\mathbf{bounding\ number}$ of $(H, <)$, denoted by bn$(H, <)$. A family $\overline{H} = ((H_a, <_a) : a \in A)$ of partial orderings is called $\mathbf{progressive}$ if $|A|^+ <$ bn$(H_a, <_a)$ for all $a \in A$. Let $f, g \in \prod \overline{H}$. If $\{a \in A : \neg(f(a) <_a g(a))\} \in I$, then we write $f <_I g$. We denote by $\check{I}$ the set $\{J \in$ Ideal$(A) : I \subseteq J\}$. Let λ be a cardinal. For the definition of $I_{<\lambda}(\tau)$ we distinguish two cases. If there is an ideal $K \in \check{I}$ such that $\lambda \leq \tau(K)$, then $I_{<\lambda}(\tau) := \bigcap\{K \in \check{I} : \lambda \leq \tau(K)\}$. Otherwise $I_{<\lambda}(\tau) :=$ $\mathcal{P}(A)$ is the power set of A. If $I = \{\emptyset\}$, then we write $J_{<\lambda}(\tau)$ instead of $\{\emptyset\}_{<\lambda}(\tau)$. $(I_{<\lambda}(\tau) : \lambda \in$CN) is called the $\mathbf{relative}$ and $(J_{<\lambda}(\tau) : \lambda \in$CN) is called the $\mathbf{absolute\ hierarchy}$. For the rest of the abstract let τ: Ideal $(A) \longrightarrow$ CN be continuous.

$\mathbf{Lemma\ 1}$ If $\lambda \in$ Range $(\tau \restriction \check{I})$, then $I_{<\lambda}(\tau) = \bigcap\{K \in \check{I} : \tau(K) = \lambda\}$.

$\mathbf{Lemma\ 2}$ (i) If $\mu, \lambda \in$ CN such that $\mu < \lambda$, then $I_{<\mu}(\tau) \subseteq I_{<\lambda}(\tau)$.
(ii) If λ is a limit cardinal, then $I_{<\lambda}(\tau) = \bigcup\{I_{<\mu}(\tau) : \mu \in$CN $\cap \lambda\}$.
(iii) max Range $(\tau \restriction \check{I}) =$ min $\{\lambda \in$ CN: $A \in I_{<\lambda^+}(\tau)\}$.

$\mathbf{Lemma\ 3}$ (St. Neumann)
1. $I_{<\lambda}(\tau) \neq \mathcal{P}(A) \Longrightarrow \lambda \leq \tau(I_{<\lambda}(\tau))$.
2. The following are equivalent:

(α) $\lambda \in$ Range $(\tau \restriction \check{I})$.
(β) $\tau(I_{<\lambda}(\tau)) = \lambda$.
(γ) $I_{<\lambda}(\tau) \subsetneqq I_{<\lambda^+}(\tau)$.

Corollary 4 $|\text{Range}(\tau)| \leq 2^{|A|}$.

Thm 5 (St. Neumann, S. Shelah) If $\overline{H}$ is a progressive family of partial orderings and bn: Ideal$(I) \to$ CN is defined by bn(I) :=bn$(\prod \overline{H}, <_I)$, then bn is a continuous ideal function.

Lemma 6 There is an example of a continuous ideal function, called the **chain-model.**

Roughly speaking we can say that Shelah's model has all properties and the chain-model has no property.

Thm 7 (St. Neumann) In Shelah's model there is, for each cardinal λ, a subset $B_\lambda \subseteq A$ such that $J_{<\lambda^+}(\tau) = J_{<\lambda}(\tau)[B_\lambda]$. In the chain-model, this is not true.

Thm 8 In Shelah's model the intersection of λ-stable ideals is λ-stable, whereas in the chain model this is not true.

A continuous ideal function τ : Ideal $A \longrightarrow$ CN is said to be **smooth** if $\tau(I \restriction B) = \tau(\mathcal{P}(B) \cap I)$ for all $B \in I^+$ and all $I \in$ Ideal(A).

Thm 9 In Shelah's model the ideal function bn is smooth, but in the chain model there are ideal functions which are not smooth.

PHILIP WELCH (Bristol)
The Inner Model Hypothesis
Sat 23.9.2006, 17:00-17:25, Zs.

S-D Friedman has introduced the Inner Model Hypothesis: "For any sentence of set theory if there is an outer model V^* (for example obtained by forcing) extending the universe V of sets (but having the same ordinals), and there is an inner model of V^* in which ϕ is true, then in fact there was already an inner model of V in which ϕ was true." It is thus a form of a maximalising principle for inner models. In joint work with Friedman, and W.H.Woodin we give some upper and lower bounds on the consistency strength of this principle, and consider some extensions.

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