Young Researchers in Set Theory Workshop 2011

21-25 March 2011, Königswinter near Bonn, Germany

Welcome to Young Set Theory 2011!

The Young Set Theory Workshop originated in Bonn in January 2008 and has since become increasingly popular. Subsequent meetings were held April 2009 near Barcelona and February 2010 near Vienna. Since the first meeting, the workshops have grown in size and scope. The Young Set Theory Workshop has been established as an important annual event in set theory and now draws many participants from Europe, North America, South America, and Asia.

The goal of this workshop is to bring together postgraduates and postdocs in set theory in order to learn from senior researchers in the field, hear about the latest research, discuss research issues in small focused groups, and give every participant the opportunity to meet and exchange ideas with fellow set theorists in their area of research. This is achieved through the unique format of tutorials, postdoc talks, and discussion sessions.

This year in particular, there is a very attractive combination of talks on topics ranging from descriptive set theory, forcing, and models of set theory to combinatorics and inner model theory.

We believe that this meeting is also a great opportunity to talk to fellow participants to find out how the job market works in various countries. We would like to encourage participants to ask people from the respective countries that they are interested in.

We wish you a fruitful and rewarding time at this conference.

Philipp Schlicht
for the organizers

In memory of Greg Hjorth (14 June 1963 - 13 January 2011)
a great friend and mathematician

Organizing Committee

Andrew Brooke-Taylor (Bristol)
Benjamin Seyfferth (Bonn)
Ioanna Dimitriou (Bonn)
Miguel Angel Mota (Vienna)
Philipp Schlicht (Bonn), chair
Thilo Weinert (Bonn)

Scientific Committee

Bart Kastermans (Boulder), chair
Daisuke Ikegami (Helsinki)
Julien Melleray (Lyon)
Marcin Sabok (Warsaw)
Sean Cox (Münster)
Thomas Johnstone (New York)
Vera Fischer (Vienna)

Contents

Schedule

Monday, March 21

8:40-10:30	Joel David Hamkins "An introduction to Boolean ultrapowers", part 1
10:30-10:50	*Coffee break*
10:50-12:40	Slawomir Solecki "Borel equivalence relations and Polish groups", part 1
12:40-13:40	*Lunch*
14:10-17:10	Discussion session with coffee at 16:00
17:10-18:00	Katie Thompson "LOTS (of) embeddability results"
18:30-19:30	*Dinner*

Tuesday, March 22

8:40-10:30	Joel David Hamkins "An introduction to Boolean ultrapowers", part 2
10:30-10:50	*Coffee break*
10:50-12:40	Slawomir Solecki "Borel equivalence relations and Polish groups", part 2
12:40-13:40	*Lunch*
14:10-17:10	Discussion session with coffee at 16:00
17:10-18:00	Assaf Rinot "Around Jensen's square principle"
18:30-19:30	*Dinner*

Wednesday, March 23

8:40-10:30	Ali Enayat "Arithmetic, set theory, and their models", part 1
10:30-10:50	*Coffee break*
10:50-11:40	Samuel Coskey "Borel equivalence relations and the conjugacy problem"
11:50-12:40	Dilip Raghavan "P-ideal dichotomy and weak squares"
12:40-13:40	*Lunch*
14:10-15:30	Discussion session
15:30-18:30	*Excursion*
18:30-19:30	*Dinner*

Thursday, March 24

8:40-10:30	Ali Enayat "Arithmetic, set theory, and their models", part 2
10:30-10:50	*Coffee break*
10:50-12:40	Juris Steprans "Unique amenability of subgroups of the symmetric group acting on the integers", part 1
12:40-13:40	*Lunch*
14:10-17:10	Discussion session with coffee at 16:00
17:10-18:00	David Schrittesser "Around supercompactness and PFA"
19:30	*Conference Dinner in Bonn*

FRIDAY, MARCH 25

8:40-10:30	Juris Steprans "Unique amenability of subgroups of the symmetric group acting on the integers", part 2
10:30-10:50	*Coffee break*
10:50-11:40	Grigor Sargsyan "An invitation to inner model theory"
11:50-12:40	Discussion session
12:40-13:40	*Lunch*
14:00	*Departure*

Abstracts

Arithmetic, set theory, and their models

Ali Enayat
American University Washington D.C., USA
enayat [at] american.edu

Wednesday and Thursday, 8:40-10:30

This tutorial will survey old and new results that illustrate the remarkable connections between set theory and models of arithmetic. Our emphasis, naturally, will be on the set-theoretical side of the story. Some longstanding open questions regarding models of arithmetic that are intimately connected with higher set theory will be included in our discussion.

Part I of the tutorial is centered on the notion of *end embeddings.* We will begin with the *McDowell-Specker-Gaifman Theorem* (dealing with models of arithmetic), which we will establish via an iterated ultrapower construction. As we shall see, this theorem has an impressive number of analogues and variants, some pertaining to models of arithmetic, and even more dealing with models of set theory (including classical results of Scott, Kunen, and Keisler-Morley, as well as fairly recent ones). Part I will conclude with a close look at the theory $\mathsf{ZFC} + \Lambda$, where Λ is the *Lévy scheme* that asserts, for each standard natural number n, the existence of an n-Mahlo cardinal κ that is n-reflective (i.e., V_κ is a Σ_n-elementary submodel of the universe).

Part II of the tutorial concerns *endomorphims* (self-embeddings). Our discussion will start with a proof of a striking result of Harvey Friedman, which states that every countable nonstandard model of PA or ZF is isomorphic to a proper (rank) initial segment of itself. We then turn to automorphisms and explain how they can be used as a 'lens' to detect *canonical* set theories, including the aforementioned

$\mathsf{ZFC} + \Lambda$. Part II includes a fairly detailed discussion of the proof of the following result:

Theorem [E, 2004].
(a) *If j is a nontrivial automorphism of a model $\mathcal{M}$ of a fragment of* ZFC (*that is even weaker than Kripke-Platek set theory*) *such that the fixed-point set of j forms a* ***rank initial segment*** *of $\mathcal{M}$, then the fixed-point set of j is a model of* $\mathsf{ZFC} + \Lambda$.

(b) *Given any consistent extension T of* $\mathsf{ZFC} + \Lambda$, *there is a model $\mathcal{M}$ of set theory* (*indeed of T*), *and a nontrivial automorphism j of $\mathcal{M}$, such that the fixed-point set of j forms a* ***rank initial segment*** *of $\mathcal{M}$ and is a model of T.*

Around Jensen's square principle

Assaf Rinot
Ben Gurion University, Be'er Sheva, Israel
assaf [at] rinot.com

Tuesday, 17:10-18:00

Jensen's square principle for a cardinal λ asserts the existence of a particular ladder system over λ^+. This principle admits a long list of applications including the existence of non-reflecting stationary sets, and the existence of particular type of trees.

In this talk, we shall be concerned with the weaker principle, *weak square*, and the stronger principle, *Ostaszewski square*, and shall study their interaction with the classical applications of the square principle.

We shall isolate a non-reflection principle that follows from weak square, and discuss tree constructions based on Ostaszewski squares. We shall present a rather surprising forcing notion that may (consistently) introduce weak square, and discuss a coloring theorem for pairs of ordinals, based on minimal walks along Ostaszewski squares.

Arithmetic, Set Theory, and their models

David Schrittesser
Rheinische Friedrich-Wilhelms-Universität Bonn, Germany
david [at] logic.univie.ac.at

Thursday, 17:10-18:00

I give a short survey on recent partial results concerning the consistency strength of PFA, such as those of Neeman and Viale and Weiß. I discuss characterizations of supercompactness in terms of reflection and "supercompactness without inaccessibility", i.e. ITP(κ) and their relation to the forcing axiom. I also present the result that ITP(ω_2) does not decide the size of the continuum.

P-ideal dichotomy and weak squares

Dilip Raghavan
Fields Institute, Toronto, Canada
raghavan [at] math.toronto.edu

Wednesday, 11:50-12:40

We answer a question of Magidor and Cummings by showing that the P ideal dichotomy of Todorčević refutes certain weak versions of the square principle. The proof uses an appropriate generalization of ρ functions.

An invitation to inner model theory

Grigor Sargsyan
University of California, Los Angeles, USA
grigor [at] math.ucla.edu

Friday, 10:50-11:40

Sometime in 1960's Scott showed that the existence of a measurable cardinal implies that $V \neq L$. L, the constructible universe, has a very canonical structure and Scott's theorem makes it impossible not to ask whether large cardinals can coexists with a kind of canonical structure given by L. Over the years, this vague question has been made precise via the introduction of extender models known as *mice*. Today it is known as the *inner model problem*.

The inner model problem: Construct extender models with large cardinals.

In this talk we will outline the progress that has been made on inner model problem and time permitting, we will also outline some of the current developments.

An introduction to Boolean ultrapowers

Joel David Hamkins
City University of New York, USA
jhamkins [at] gc.cuny.edu

Monday and Tuesday, 8:40-10:30

Boolean ultrapowers generalize the classical ultrapower construction on a power-set algebra to the context of an ultrafilter on an arbitrary complete Boolean algebra. Closely related to forcing and particularly to the use of Boolean-valued models in forcing, Boolean ultrapowers were introduced by Vopěnka in order to carry out forcing as an internal ZFC construction, rather than as a meta-theoretic argument as in Cohen's approach. An emerging interest in Boolean ultrapowers has arisen from a focus on the well-founded Boolean ultrapowers as large cardinal embeddings.

Historically, researchers have come to the Boolean ultrapower concept from two directions, from set theory and from model theory. Exemplifying the set-theoretic perspective, Bell's classic (1985) exposition emphasizes the Boolean-valued model $V^{\mathbb{B}}$ and its quotients $V^{\mathbb{B}}/U$, rather than the Boolean ultrapower V_U itself, which is not considered there. Mansfield (1970), in contrast, gives a purely algebraic, forcing-free account of the Boolean ultrapower, emphasizing its potential as a model-theoretic technique, while lacking the accompanying generic objects.

The unifying view I will explore in this tutorial is that the well-founded Boolean ultrapowers reveal the two central concerns of set-theoretic research—forcing and large cardinals—to be two aspects of a single underlying construction, the Boolean ultrapower, whose consequent close connections might be more fruitfully explored. I will provide a thorough introduction to the Boolean ultrapower construction, while assuming only an elementary graduate student-level familiarity with set theory and the classical ultrapower and forcing techniques.

Unique amenability of subgroups of the symmetric group acting on the integers

Juris Steprans
York University, Toronto, Canada
steprans [at] yorku.ca

Thursday 10:50-12:40, and Friday 8:40-10:30

In an article published in 1994 in the *Journal of Functional Analysis* — Volume 126, pages 7 to 25, to be precise — Matt Foreman constructed an amenable, discrete group with a unique amenable action on the integers assuming various set theoretic hypotheses such as $\mathfrak{p} = \mathfrak{c}$ or $\mathfrak{u} = \aleph_1$. This tutorial will introduce amenability, provide the motivation for Foreman's result, explain the necessary set theoretic notions and describe most of the details of Foreman's arguments. The goal of the talks will be to supply those interested with enough background to begin working on the outstanding open questions in this area at the juncture of abstract harmonic analysis and set theory.

LOTS (of) embeddability results

Katie Thompson
Vienna University of Technology, Austria
thompson [at] logic.univie.ac.at

Monday, 17:10-18:00

A linearly ordered topological space or LOTS is a linear order equipped with the open interval topology. LOTS embeddings are a natural extension of linear order embeddings, i.e. injective and order-preserving, which are also continuous. In joint work with Alex Primavesi, we show that the addition of continuity has strong effects on the embedding structure. In particular, there is no known model of set theory which has a universal LOTS at any uncountable cardinal, however, there are restricted classes of LOTS where there are universals. Also remarkably, under PFA there is an 11 element basis for the uncountable LOTS, which extends the 5 element basis for linear orders proved by J. Moore.

Borel equivalence relations and the conjugacy problem

Samuel Coskey
Fields Institute, Toronto, Canada
scoskey [at] nylogic.org

Wednesday, 10:50-11:40

One of the most interesting classification problems in mathematics is the *conjugacy problem* for a given group. Indeed, when studying a finite group it is usual to write down its class equation and study the set of conjugacy classes. For larger groups (say Polish groups) one similarly should study the conjugacy equivalence relation.

The idea of this talk is to isolate the complexity of the conjugacy relation on a few special groups. The relevant notion of complexity comes from an area of study called Borel equivalence relations. Here, if E, F are equivalence relations on (standard) Borel spaces X, Y, we say that E is *Borel reducible* to F iff there exists a Borel function $f\colon X \to Y$ such that

$$x \; E \; x' \iff f(x) \; F \; f(x') \; .$$

I will give a short introduction to the theory of Borel reducibility, and then use this tool to analyze the conjugacy problem for some of the most famous and well-studied groups in logic: the automorphism groups of $\mathbb{Q}$, the random graph, and other ultrahomogeneous structures.

Borel equivalence relations and Polish groups

Sławomir Solecki
University of Illinois at Urbana-Champaign
ssolecki [at] math.uiuc.edu

Monday and Tuesday, 10:50-12:40

In many places in mathematics, important spaces are quotients of Polish spaces by Borel equivalence relations. In general, such quotients do not carry natural reasonable topologies or even Borel structures. One can, however, study them by exploring the equivalence relation used to form the quotient. Descriptive Set Theory provides tools for this exploration. In the tutorial, I will present basic such tools, and then I will concentrate on the roughest conjectural division of the class of Borel equivalence relations: each Borel equivalence relation is to be either Borel reducible to the orbit equivalence relation of a continuous action of a Polish group, or otherwise is to Borel reduce a complicated Borel equivalence relation called E_1. This dichotomy has been proved so far only in very restricted contexts. I will present the known results on this topic. This will lead us to the notion of Polishable group. I will present descriptive set theoretic structure of such groups and describe connections of this structure with other mathematical properties of groups.

All the material in my tutorial will come from the following papers.

1. A.S. Kechris, A. Louveau, *The structure of hypersmooth equivalence relations*, J. Amer. Math. Soc. 10 (1997), 215–242.
2. S. Solecki, *Polish group topologies*, in *Sets and Proofs*, pp. 339–364, London Math. Soc. Lecture Note Ser. 258, Cambridge Univ. Press, 1999.
3. I. Farah, S. Solecki, *Borel subgroups of Polish groups*, Adv. Math. 199 (2006), 499–541.
4. S. Solecki, *The coset equivalence relations and topologies on subgroups*, Amer. J. Math. 131 (2009), 571–605.

Research statements

The research statements are ordered alphabetically by first name.

Alexander Primavesi
University of East Anglia, Norwich, UK
a.primavesi [at] uea.ac.uk

I began my doctoral research by looking at a long-standing question of the set theorist I. Juhasz, of whether the axiom $\clubsuit$ implies the existence of a Suslin tree (an uncountable tree with no uncountable chains or antichains). The existence of Suslin trees is known to be independent of ZFC. $\clubsuit$ is one of a family of axioms known as 'guessing axioms' and is a natural weakening of $\diamondsuit$, which is itself a strengthening of the Continuum Hypothesis and is known to imply the existence of a Suslin tree. $\clubsuit$ + CH is equivalent to $\diamondsuit$, so $\clubsuit$ can be thought of as $\diamondsuit$ without the cardinal arithmetic assumptions. Juhasz's question is one of a class of natural questions that ask: how different are $\diamondsuit$ and $\clubsuit$?

My research is concerned with several such questions. Two examples are the following:

- $\diamondsuit$ has an *invariance property* in the sense that making small changes to its definition won't, in general, get you a different (either strictly stronger or strictly weaker) statement. To what extent is this invariance property shared by $\clubsuit$?
- It is known that $\clubsuit$ is consistent with $\neg$CH, but there are related questions that remain unanswered. For example, is it possible to force $\clubsuit$ from a model of $\neg$CH without collapsing 2^ω?

These are questions that came to light in my research into Juhasz's question and could have application in answering it. The definition of $\clubsuit$ is as follows:

Definition. ($\clubsuit$) *There is a sequence* $\langle A_\delta : \delta \in Lim(\omega_1)\rangle$ *such that* $A_\delta \subseteq \delta$ *and* $\sup(A_\delta) = \delta$*, and if* $X \subseteq \omega_1$ *is uncountable then the set* $\{\delta < \omega_1 : A_\delta \subseteq X\}$ *is stationary.*

Ali Enayat
American University, Washington DC, USA
enayat [at] american.edu

My research is principally focused on *foundational* axiomatic systems ranging in strength from weak theories of arithmetic in which exponentiation of finite numbers might be a partial operation, all the way up to systems of set theory with large cardinals. I am also interested in 'alternative set theories' (e.g., Quine-Jensen set theory NFU) and their precise relation - at the interpretability level - with orthodox ZF-style theories of sets. A common theme in my work is the use of a combination of model-theoretic and set-theoretic methods and ideas to gain a better understanding of the complex web of affinities and disparities between finite and infinite set theory.

Andrea Medini
University of Wisconsin-Madison Mathematics Department, USA
medini [at] math.wisc.edu

My research is in Set-Theoretic/General Topology. More specifically, I have been working on h-homogeneity, CLP-compactness and their behaviour under products. A general fact that contributes to making those topics interesting is that clopen subsets of products need not be the union of clopen rectangles (see [2]).

A topological space X is *h-homogeneous* if all non-empty clopen subsets of X are homeomorphic (to X). The Cantor set, the rationals, the irrationals or any connected space are examples of h-homogeneous spaces. In [7], building on work of Terada (see [12]) and using Glicksberg's classical theorem on the Stone-Čech compactification of products, I obtained the following result.

Theorem. *Assume that X_i is zero-dimensional and h-homogeneous for every $i \in I$. Then $X = \prod_{i \in I} X_i$ is h-homogeneous.*

Furthermore, if X is pseudocompact, then the zero-dimensionality requirement can be dropped. (I don't know whether the zero-dimensionality requirement can be dropped in general.) Along the way, I showed that clopen subsets of pseudocompact products depend only on finitely many coordinates, thus generalizing a result of Broverman (see [1]). Also, I gave some partial answers to the following question from [12], which remains open.

Question (Terada)**.** *Is X^ω h-homogeneous whenever X is zero-dimensional and first-countable?*

If one drops the 'h', then the answer is 'yes' by a remarkable theorem of Dow and Pearl (see [4]). Since h-homogeneity implies homogeneity for zero-dimensional first-countable spaces, a positive answer would give a strenghtening of their result. For other interesting papers on h-homogeneity, see [3], [5], [8], [9] or [13].

A topological space X is *CLP-compact* if every cover of X consisting of clopen sets has a finite subcover. For zero-dimensional spaces, CLP-compact-ness is the same as compactness. In [6], I obtained the following result, which answers a question of Steprāns and Šostak from [11]. The proof involves the construction of a special family of finite subsets of ω^*.

Theorem. *For every infinite cardinal κ, there exists a family $\{X_\xi : \xi \in \kappa\}$ such that $\prod_{\xi \in F} X_\xi$ is CLP-compact for every $F \in [\kappa]^{<\omega}$ while $\prod_{\xi \in \kappa} X_\xi$ is not.*

For a positive result on (finite) products of CLP-compact spaces, see [10].

References

[1] S. Broverman. The structure of continuous $\{0,1\}$-valued functions on a topological product. *Canad. J. Math.* **28:3** (1976), 553–559.

[2] R. Z. Buzyakova. On clopen sets in Cartesian products. *Comment. Math. Univ. Carolin.* **42:2** (2001), 357–362.

[3] E. K. van Douwen. A compact space with a measure that knows which sets are homeomorphic. *Adv. in Math.* **52:1** (1984), 1–33.

[4] A. Dow and E. Pearl. Homogeneity in powers of zero-dimensional first-countable spaces. *Proc. Amer. Math. Soc.* **125** (1997), 2503–2510.

[5] M. V. Matveev. Basic homogeneity in the class of zero-dimensional spaces. *Filomat.* **12:2** (1998), 13–20.

[6] A. Medini. A non-CLP-compact product space whose finite subproducts are CLP-compact. *Topology Appl.* **157:18** (2010), 2829–2833. (Also available on `http://arxiv.org`.)

[7] A. Medini. Products and h-homogeneity. *Topology Appl.* (Accepted, available on `http://arxiv.org`.)

[8] D. B. Motorov. Homogeneity and π-networks. *Vestnik Moskov. Univ. Ser. I Mat. Mekh.* **44:4** (1989), 31–34 (in Russian); English translation in: *Moscow Univ. Math. Bull.* **44:4** (1989), 45–50.

[9] S. Shelah and S. Geschke. Some notes concerning the homogeneity of Boolean algebras and Boolean spaces. *Topology Appl.* **133:3** (2003), 241–253.

[10] J. Steprāns. Products of sequential CLP-compact spaces are CLP-compact. *Ann. Pure Appl. Logic* **143:1-3** (2006), 155–157.

[11] J. Steprāns and A. Šostak. Restricted compactness properties and their preservation under products. *Topology Appl.* **101:3** (2000), 213–229.

[12] T. Terada. Spaces whose all nonempty clopen subsets are homeomorphic. *Yokohama Math. Jour.* **40** (1993), 87–93.

[13] R. de la Vega. Basic homogeneity in the class of zero-dimensional compact spaces. *Topology Appl.* **155:4** (2008), 225–232.

Andrew Brooke-Taylor

University of Bristol, UK

`andrew.brooke-taylor [at] bristol.ac.uk`

My research centres around large cardinal axioms and class forcing, although I am also very interested in applications of set theory, and I have also been working on Fraïssé limits.

With Sy Friedman I have been working on the interaction between large cardinals and $\square$: we show that if some κ is α^+*-subcompact*, then $\square_\alpha$ must fail, but we can force $\square$ to hold everywhere else. Similar results hold for stationary reflection. With Joan Bagaria, I have been working on applications of large cardinal axioms in category theory and related areas. I have also been working with Benedikt Löwe and Birgit Richter on set-theoretic properties of the *Bousfield lattice*, an important construct in algebraic topology. I have a paper with Damiano Testa that we've been trying to finish for a couple of years now about Fraïssé limits for infinite relational languages that locally behave like finite ones. I am also interested in the connections between rank-to-rank embeddings and LD-systems (in algebra), which I got into working on with Sheila Miller back at the first Young Set Theory Workshop.

Assaf Rinot
Ben-Gurion university, Be'er Sheva, Israel
`assaf [at] rinot.com`

I am a post-doc at the center for advanced studies in mathematics, at Ben-Gurion university, Be'er Sheva, Israel.

My research focuses on combinatorial set theory. More specifically, my main interest is the study of the combinatorics of concepts which were first discovered to hold in Gödel's constructible universe.

Recently, we introduced a few variations of Jensen's square principle, which we dub as *Ostaszewski's squares.* First, recall the original notion: $\square_\kappa$ asserts the existence of a sequence $\overrightarrow{C} = \langle C_\alpha \mid \alpha < \kappa^+\rangle$ such that C_α is a club in α of type $\leq \kappa$, and for which $C_\beta = C_\alpha \cap \beta$ whenever $\sup(C_\alpha \cap \beta) = \beta$. Next, we say that $\overrightarrow{C}$ is an Ostaszewski $\square_\kappa$-sequence, if, in addition, for every limit $\theta < \kappa$, every club $D \subseteq \kappa^+$, and every unbounded $A \subseteq \kappa^+$, there exists some $\alpha < \kappa^+$ for which all of the following holds:

(1) $\mathrm{otp}(C_\alpha) = \theta$;
(2) $\mathrm{acc}(C_\alpha) \subseteq D$;

(3) $\mathrm{nacc}(C_\alpha) \subseteq A$.

Here, $\mathrm{acc}(C_\alpha)$ stands for the set $\{\beta \in C_\alpha \mid \sup(C_\alpha \cap \beta) = \beta\}$, and $\mathrm{nacc}(C_\alpha)$ stands for $\{\beta \in C_\alpha \mid \sup(C_\alpha \cap \beta) \neq \beta\}$.

We've been studying the behavior of Todorcevic's minimal walks along such sequences, and also reformulated many of the constructions of higher Souslin trees, as applications of Ostaszewski's squares. Finally, our main recent result is that the Ostaszewski square follows from the usual square, assuming fragments of GCH.

Brent Cody
City University of New York, USA
`bcody [at] gc.cuny.edu`

My research is in the field of set theory, more specifically large cardinals and forcing. I have mostly been concerned with the interaction between GCH and large cardinals.

Scott proved that GCH cannot first fail at a measurable cardinal and it has become typical to expect that properties of a measurable cardinal κ will reflect to a measure one set below κ. I am currently working with Arthur Apter on showing that in many diverse models, GCH may hold at a measurable cardinal κ and yet fail at every regular below κ.

Woodin showed that the existence of a measurable cardinal at which GCH fails is equiconsistent with the existence of a cardinal κ that is κ^{++}-tall, where a cardinal κ is called *θ-tall* if there is a nontrivial elementary embedding $j : V \to M$ with critical point κ such that $j(\kappa) > \theta$ and $M^\kappa \subseteq M$ in V.

I have extended Woodin's method of surgically modifying a generic filter to the context of supercompactness embeddings. I have used this method to determine the precise consistency strength of the existence of a λ-supercompact cardinal κ such that GCH fails at λ.

Theorem. *The existence of a λ-supercompact cardinal κ such that $2^\lambda \geq \theta$ is equiconsistent with the existence of a λ-supercompact cardinal that is also θ-tall.*

In my dissertation I will determine which GCH patterns can consistently arise on the regular cardinals alongside a λ-supercompact cardinal κ, starting with an additional large cardinal hypothesis on κ.

Carolin Antos-Kuby
Kurt Gödel Research Center, University of Vienna, Austria
caro.antos-kuby [at] punktrot.de

I am in the first year of my PhD working under the supervision of Sy D. Friedman. Beginning with my master thesis, which was concerned with work of Joel Hamkins about extensions which do not create new large cardinals, I became interested in Hamkins' research about set-theoretic geology (for an introduction see [1]). Here, the structural relations between the universe V and its inner models are investigated by looking at the universe as a (set) forcing extension of some ground model. A fundamental result is a theorem by Laver which states that every model of set theory is a definable class in all of its set forcing extensions, using parameters from this model (see [2]). Jonas Reitz [3] used this theorem to introduce two new axioms: the Ground Axiom, stating that V is not the nontrivial set forcing extension of any inner model, and the Bedrock Axiom, stating that there is an inner model W, such that V is a set forcing extension of W and W is a model of the Ground Axiom. Consequently, a transitive class W is defined to be a ground of V if $W \models ZFC$ and $V = W[G]$ is a forcing extension of W by set forcing $G \subseteq P \in W$. If in addition there is no deeper ground inside the ground W, then W is called a bedrock of V. There are several open questions in this field, for example: Is the bedrock of a model unique, when it exists? Are the grounds downward directed? There might be the possibility to approach these questions by using the following result of Bukovsky from 1973 (see [4]), which was recently rediscovered by Sy Friedman: The ground models of V are exactly the inner models M with the property that M globally covers V, which means that for some V-regular κ, if $f : \alpha \to M$ belongs to V, then there is $g : \alpha \to M$ in M such that $f(i) \in g(i)$ and $g(i)$ has V-cardinality $< \kappa$ for all $i < \alpha$.

As the above results and questions are restricted to set forcing, one goal of my doctoral thesis will be to extend them to class forcing.

References

[1] J. Hamkins *Some second order set theory*, in R. Ramanujam and S. Sarukkai, editors, ICLA 2009, volume 5378 of LNAI, 36–50. Springer-Verlag, Amsterdam, 2009.
[2] R. Laver *Certain very large cardinals are not created in small forcing extensions*, Annals of Pure and Applied Logic 149 (2007), 1–6.
[3] J. Reitz *The ground axiom*, Journal of Symbolic Logic 72 (2007), no.4, 1299–1317.
[4] L. Bukovsky *Characterization of generic extensions of models of set theory*, Fund. Math. 83 (1973), 35–46.

Carsten Szardenings
University of Münster, Germany
`CSzardenings [at] gmx.de`

My main interests lie in the interplay of set theory and other branches of mathematics, specifically algebra, topology, ergodic theory. More precisely:

My diploma thesis dealt with bounding the height of automorphism towers. Given a Group G with trivial center, its automorphism Group $Aut(G)$ has trivial center as well. Furthermore there exists a natural embedding from G to $Aut(G)$ mapping an element $g \in G$ to the conjugation with g. By identifying G with its image under that embedding, setting $G_0 = G$ and $G_1 = Aut(G)$ and iterating this process while setting $G_\alpha = \bigcup_{\gamma<\alpha} G_\gamma$ at limit stages, we obtain an ascending chain of groups, the automorphism tower of G.
If $\alpha \in \mathbf{On}$ is minimal with $G_\alpha = G_{\alpha+1}$ we call α the height of the automorphism tower of G (denoted τ_G). Let τ_κ be the least

upper bound for the height of automorphism towers of groups with cardinality κ. Itay Kaplan and Saharon Shelah showed that in ZF $\tau_\kappa < \theta_{\mathcal{P}(^{<\omega}\kappa)}$, where $\theta_{\mathcal{P}(^{<\omega}\kappa)}$ denotes the minimal ordinal such that there is no surjection from $\mathcal{P}(^{<\omega}\kappa)$ to it. In the presence of AC this results in the well-known inequality $\tau_\kappa < (2^\kappa)^+$. Itay Kaplan and Saharon Shelah showed also that $(\tau_\kappa)^{V'} = (\tau_\kappa)^V$ for a transitive class model $V' \subseteq V$ of ZF with $\mathcal{P}(\kappa) \in V'$. On the other hand following an approach of Winfried Just, Saharon Shelah and Simon Thomas one can use forcing to show that the bound $\tau_\kappa < (2^\kappa)^+$ is the best cardinal bound provable in ZFC.

I'm still interested in questions concerning the function that maps κ to τ_κ or if better bounds are provable once you restrict yourself to certain classes of groups, however in the recent months, i.e. since I am a PhD student, my main focus has lain on descriptive set theory, particularly the theory of definable/Borel equivalence relations on Polish spaces. This yields applications to classification problems in ergodic theory, since given a standard measure space (X, μ) (any such space is isomorphic to the interval $[0, 1]$ with the Lebesgue measure) and a measure preserving (or ergodic) transformation T the orbit equivalence relation E_T, where for $x, y \in X$ xE_Ty iff there exists an $n \in \mathbb{Z}$ such that $x = T^n(y)$, is a Borel equivalence relation. The study of those equivalence relations is linked to the study of Borel actions of countable groups on Polish spaces, which is an interesting area on its own.

Christoph Weiss
University of Munich, Germany
`fiesh [at] fiesh.homeip.net`

Recently, Matteo Viale and I proved that producing a model of PFA using a standard forcing iteration requires a strongly compact cardinal. If the forcing is proper, then a supercompact cardinal is necessary. These results rely on the principles TP and ITP from my thesis. They characterize strong compactness and supercompactness for inaccessible cardinals but can consistently hold for small cardinals. The proof works by showing PFA implies the principles hold for ω_2 and then pulling them back to the ground model. Key to this pulling back are the covering and approximation properties.

It seems plausible these ideas might be of help in the attempt to build an inner model with a supercompact cardinal from PFA. This is my current interest in research, and I hope that it will at least shed some new light from a different perspective on the, in my opinion, most important open problem in set theory.

Daisuke Ikegami
University of Helsinki, Finland
daiske.ikegami [at] gmail.com

My interest in set theory is **descriptive set theory**, especially determinacy, forcing absoluteness, and their connections with large cardinals and inner model theory. Currently, I am mainly working on **Blackwell determinacy** and its connection with **Gale-Stewart games**. Blackwell games are infinite games with imperfect information generalizing the game "Rock-Paper-Scissors" and Blackwell determinacy is an extension of von Neumann's minimax theorem for Blackwell games while Gale-Stewart games are infinite games with perfect information generalizing the game "Chess" and the determinacy of Gale-Stewart games has been deeply investigated in set theory.

In 1998, Martin proved that the Axiom of Determinacy (AD) implies the Axiom of Blackwell determinacy (Bl-AD) and conjectured the converse, which is still open to be true. In 2003, Martin, Neeman, and Vervoort proved that AD and Bl-AD are equiconsistent. Recently, with de Kloet and Löwe, I introduced the Axiom of Real Blackwell determinacy (Bl-$\mathrm{AD}_{\mathbb{R}}$) and proved that Bl-$\mathrm{AD}_{\mathbb{R}}$ implies the consistency of AD, so by Gödel's Incompleteness Theorem, the consistency of Bl-$\mathrm{AD}_{\mathbb{R}}$ is strictly stronger than that of AD.

Currently I am working with Woodin on the connection between Bl-$AD_{\mathbb{R}}$ and the Axiom of Real Determinacy ($AD_{\mathbb{R}}$). We are about to prove that they are equivalent assuming the Axiom of Dependent Choice (DC) and are working on whether they are equiconsistent. (Note that $AD_{\mathbb{R}}$+DC implies the consistency of $AD_{\mathbb{R}}$ by the result of Solovay. So the equivalence of $AD_{\mathbb{R}}$ and Bl-$AD_{\mathbb{R}}$ under DC does not give us the equiconsistency between them.)

Apart from Blackwell determinacy, I am interested in higher forcing absoluteness (Σ^2_n forcing absoluteness for a natural number n), descriptive set theory in $\mathcal{H}_{\omega_2}$, and the inner models constructed from first-order logics with generalized quantifiers (those obtained like L by replacing "first-order definable sets" by "definable sets by the logics").

Daniel Donado
Universidad de los Andes, Bogota, Colombia
d-donado [at] uniandes.edu.co

I have just finished my thesis for degree of Magister in Mathematics at the Universidad de los Andes working under directions of Andres Caicedo from Boise State (Idaho), about clasification of metric spaces in ω_1 under Axiom of Determinacy and $V = L(\mathbb{R})$; following the doctoral dissertation of Apollo Hogan (2004). My research is focused on improve the classification of ordered metric spaces, studying more topological consecuences of Determinacy and partition properties on ω_1 and ω_2 in order to get results there and hopefully for higher cardinalities. I have encountered many issues in this goal because of the necesary tools from the theory of sharps and definability of sets that are used in the clasification on ω_1, so, looking for tools to do a similar work on a bigger cardinal, I have been recently following a seminar on Proper Forcing leaded by David Aspero and Andres Villaveces at the Universidad Nacional in Bogotá (Colombia). We have studied generalities and consecuences of the Proper Forcing Axiom and the Mapping Reflection Principle, but we have just started and some open problems that professor Aspero has exposed to us really seem interesting and motivating.

Here at Bogotá we are beginning to create a research group in set theory, but still we don´t have a solid basis, so I think this is a great oportunity to focus my research and share the iniciative we have here with a lot of excellent colleagues, making bonds with other researchers and their universities. Many students comment that this is the best oportunity to do this and I don´t want to miss it.

Dániel Tamás Soukup
Eötvös Loránd University, Hungary
`daniel.t.soukup [at] gmail.com`

I am a second year MSc student at Eötvös Loránd University, Hungary. I am mainly interested in set theoretical topology. Recently, I have been investigating the connections between D-spaces and covering properties.

Definition (E. van Douwen)**.** *A space X is said to be a D-*space *(or has property D) iff for every open neighborhood assignment U, one can find a closed discrete $D \subseteq X$ such that $X = \bigcup_{d \in D} U(d) = \bigcup U[D]$.*

I recommend G. Gruenhage's survey on D-spaces [3], which summarizes the facts and the work done in the topic, stating numerous fascinating open problems. One of the main problems with D-spaces, is that we lack theorems stating that a classical covering property weaker than compactness implies property D. As Gruenhage says, "... it is not known if a very strong covering property such as hereditarily Lindelöf implies D, and yet for all we know it could be that a very weak covering property such as submetacompact or submetalindelöf implies D!"

In a joint work with Xu Yuming [7], we examined the D-property of some generalized metric spaces: generalized stratifiable spaces, elastic spaces and the Collins-Roscoe mechanism.

Investigating D-spaces, Arhangel'skii introduced the class of aD-spaces.

Definition (Arhangel'skii, [1]). *A space X is said to be* aD *iff for each closed $F \subseteq X$ and for each open cover $\mathcal{U}$ of X there is a closed discrete $D \subseteq F$ and $N : D \to \mathcal{U}$ with $x \in N(x)$ such that $F \subseteq \bigcup N[D]$.*

Interestingly, aD-spaces are much more docile than D-spaces; Arhangel'skii showed, that every submetalindelöf space is aD [2]. Answering a question of Arhangel'skii [2], I proved that there exists an aD, non D-space [6]; the counterexamples use Shelah's club guessing theory. Nevertheless, the questions about main covering properties and D-spaces remain open.

In [5], I answered questions raised by Guo and Junnila [4] concerning characterization of linearly D-spaces; that is, in the definition of D-spaces, we only consider monotone neighborhood assignments. Also, I proved that the existence of certain "locally nice" aD, non D-spaces is independent.

Now, I am interested in getting a better insight on non D-spaces, which are linearly D and aD. I hope, that this will shed some light on the question, whether every Lindelöf space is D.

References

[1] A.V.Arhangel'skii and R. Buzyakova, Addition theorems and D-spaces, Comment. Mat. Univ. Car. 43(2002), 653-663.

[2] A.V. Arhangel'skii, D-spaces and covering properties, Topology and Appl. 146-147(2005), 437-449.

[3] G. Gruenhage, A survey of D-spaces, to appear. (http://www.auburn.edu/∼gruengf/papers/dsurv5.pdf)

[4] H. Guo and H.J.K. Junnila, On spaces which are linearly D, Topology and Appl., Volume 157, Issue 1, 1 January 2010, Pages 102-107.

[5] D. T. Soukup, Properties D and aD Are Different, Top. Proc. 38 (2011) pp. 279-299.

[6] D. T. Soukup, Guessing clubs for aD, non D-spaces, submitted to Topology and its Applications.

[7] D. T. Soukup, Xu Yuming, The Collins-Roscoe mechanism and D-spaces, accepted at Acta Mathematica Hungarica.

David Chodounsky
Charles University Prague, Czech Republic
david.chodounsky [at] matfyz.cz

The topological space consisting of free ultrafilters on a cardinal κ is denoted κ^*. It is not hard to prove that if $\kappa \neq \lambda$ and $\{\kappa, \lambda\} \neq \{\omega, \omega_1\}$ then κ^* and λ^* are not homeomorphic. The remaining case is still open. This problem can be also formulated in the language of Boolean algebras: Can $P(\omega)/Fin$ be homeomorphic to $P(\omega_1)/Fin$?

So far there are known only few non trivial consequences of existence of such homeomorphism. Namely $d = \omega_1$ and the existence of a strong Q-sequence of size ω_1 (also called uniformizable AD-system). Both of these facts are consistent with ZFC but it has not been shown yet, that they can be realized in the same model at once. (Update: A minor modification of a forcing notion from [4] provides such model.)

My aim is to use forcing methods to build models containing at least a partial approximation of such homeomorphism and also to build a model, where both conditions mentioned above are realized. During this process some other consequences of such homeomorphism may be discovered.

References

[1] . Nyikos, *Cech-Stone remainders of discrete spaces*, in: Open Problems in Topology II, Elliott Pearl, ed., Elsevier B.V., Amsterdam, 2007, 207–216.

[2] . Steprans *Strong Q-sequences and variations on Martin's Axiom*, Canad. J. Math. **37** (1985) no. 4, 730–746.

[3] . W. Comfort, *Compactifications: recent results from several countries*, Topology Proc. **2** (1977), no. 1, 61–87 (1978).

[4] H. Judah, S. Shelah, *Q-sets, Sierpiński sets, and rapid filters*, Proc. Amer. Math. Soc. **111** (1991) 821–832.

David J. Fernández Bretón
York University, Toronto, Canada
davidfb [at] mathstat.yorku.ca

My main interests are Logic and Set Theory, including Forcing and Semantics for Higher Order Logics. In the past couple of years, I have been thinking/reading/studying about Shelah's solution to Whitehead problem, as well as the new set-theoretical problems and questions that solution gave rise to, such as weak diamonds, generalizations to modules over rings of several cardinalities, etc. During that same period I also spent some time analyzing several results concerning the infinite symmetric group (the group of permutations of ω) and a couple of cardinal invariants defined from this group, focusing on upper and lower bounds for these cardinals.

More as a hobby, I like to look at alternative axiomatizations of set theory, such as NFU, or the ill-founded sets universe of Peter Aczel. I haven't still been serious about this, but I'm planning to really get into it.

Recently my supervisor, Juris Steprāns, suggested to look for some problems concerning the Borel conjecture and the notion of a Strong Measure Zero set, so I am starting to familiarize myself with the needed notions and main results on that particular topic.

Finally, I just started looking at a paper on Infinite Time Turing machines. I am really excited about this, which seems really interesting to me, and I guess I will think a great deal about this. I am trying to find a problem to work on regarding this particular topic.

David Schrittesser
Rheinische Friedrich-Wilhelms-Universität Bonn, Germany
david [at] logic.univie.ac.at

My research centers around topics in descriptive set theory and the theory of iterated forcing on the one hand, and on forcing axioms and large cardinals on the other hand.

In my Ph.D. thesis [Sch10], I investigated regularity properties of the two ideals $\mathcal{M}$ (meager) and $\mathcal{N}$ (null).

Theorem ([Sch10]). *Given the consistency strength of a Mahlo, there is a model where all projective sets are Lebesgue measurable but there is a Δ^1_3 set without the Baire property.*

In the next two years, I plan to do research in the area of cardinal characteristics on ω. Not assuming CH, it becomes an interesting field of investigation to obtain and study models where these characteristics have different cardinality.

If we strive to find models where $\mathfrak{c} = \omega_2$, many techniques are known for forcing different values for different characteristics (for example, [RS99] and [She98]). Alternatively, if we want models for $\mathfrak{c} = \omega_3$ then virtually any question involving a relation between three characteristics (obeying the usual known restraints such as the ones in Cichoń's diagram) is open, e.g.:

Question. *Construct a model where* $\mathrm{cov}(\mathcal{M}) = \omega_1$*,* $\mathfrak{d} = \omega_2$ *and* $\mathfrak{c} = \omega_3$*.*

I'm currently cooperating with Stefan Geschke to solve the following question:

Question. *Find a model where* $\mathfrak{c} = \omega_3$ *and* $\mathfrak{hm} = \omega_2$*. Possibly also consider* $\mathrm{cov}(\mathcal{M}) = \omega_1$*.*

One approach to this question would be to start with a carefully chosen model and add ω_3 many Cohen reals.

Lately Aspero and Mota [AM] have found a way of iterating proper forcing of size ω_1 for length ω_3 using *elementary submodels as side conditions*, an idea which was introduced and investigated by Todorčević. Their work may offer a blue-print how to deal with other iterations in a slightly more general manner, in contrast to the ad-hoc approaches that have been employed in this field so far, possibly by allowing side conditions both of size ω_1 and of size ω as was done recently by Italy Neeman.

The axioms $\mathbf{OCA}_{[\mathrm{ARS}]}$ and $\mathbf{OCA}_{[\mathrm{T}]}$ demand that certain homogeneous sets for open colorings of certain Polish spaces exist. Both have numerous applications, and both are consequences of PFA (but not equiconsistent with PFA). In a spectacular result Moore [Moo02] has shown that together, they imply $2^\omega = \omega_2$.

Similar approaches as the ones described above are also interesting for the following problem:

Question. *Is* $\mathbf{OCA}_{[\mathrm{ARS}]}$ *consistent with* $\mathrm{unif}(\mathcal{M}) = \omega_1$*?*

In my master's thesis I investigated the forcing axiom $\mathrm{FA}(\mathbf{\Sigma}^1_3, \Gamma)$, that is

$$\forall P \in \Gamma \quad V \prec_{\mathbf{\Sigma}^1_3} V^P,$$

for various classes of Γ, with respect to their consistency strength, e.g. obtaining:

Theorem ([Sch04], [Sch07]). $\mathrm{FA}(\mathbf{\Sigma}^1_3, \mathrm{ccc})$ *together with "ω_1 is inaccessible to reals" is equiconsistent with the existence of a* lightface Σ^1_2-indescribable *cardinal.*

To see this, one associates an Aronszajn tree to a hypothetical failure of Σ^1_2 reflection in L. The rest of the argument combines a coding technique from [HS85] with an argument very similar to the classical one showing that the tree property holds at ω_2 under PFA.

Similar ideas are elaborated by Italy Neeman and Ernest Schimmerling (see especially [Nee08]). In particular, Italy Neeman uses a morass-like construction to obtain a similar higher order reflection principle from PFA:

Theorem. *Assuming V is a proper forcing extension of an L-like model W and* PFA *holds. Then there is a Σ^2_1-indescribable 1-gap $[\kappa, \kappa^+]$ in W.*

The assumption of an L-like model was made plausible by recent work of Sy Friedman and Peter Holy [HF], who showed that any model of set theory has an extension which is L-like (i.e. satisfies a strong form of condensation).

Unfortunately it is very unclear how to obtain larger gaps; obtaining Σ^2_1-indescribable gaps $[\kappa, \lambda]$ for every $\lambda \geq \kappa$ would yield $W \models \kappa$ is supercompact.

A different approach was taken by Viale and Weiß. In his thesis, Weißhas isolated a combinatorial property of supercompact cardinals, the *list property* or ITP(κ), which can be subtracted from inaccessibility, in the sense that ITP(ω_2) can hold—and it does hold under PFA ([Wei10], [Wei]).

On the other hand, as conjectured by Viale and Weiß, ITP(ω_2) does not have as startling consequences as PFA. For example, in a joint paper with Shelah, it is proved that ITP(ω_2) is consistent with arbitrarily large continuum (assuming a supercompact cardinal; see our forthcoming [SS]). I also plan to generalize this work to larger cardinals and prove e.g. the relative consistency of ITP(ω_3) and $2^{\omega_2} > \omega_3$.

Viale and Weiß show:

Theorem ([VW]). *If W is a proper forcing extension of V by a standard iteration of length κ, where κ is inaccessible and in V and $W \models$ PFA and $\kappa = \omega_2$. Then κ is supercompact in V.*

Question. *Can you drop some of the assumptions from theorem ?*

In [Bag], Bagaria strengthens a classical characterization of supercompactness in terms of higher order reflection, or Löweinheim Skolem type properties. In a joint project Bagaria and I plan to investigate if this characterization can be used to answer question .

References

[AM] D. Aspero and M. A. Mota, *Forcing consequences of* PFA *together with the continuum large*, preprint available at `http://bcnsets.ub.es/researchers/David.Aspero/`.

[Bag] J. Bagaria, *$C(n)$ cardinals*, preprint available at `http://bcnsets.ub.es/researchers/Joan.Bagaria/`.

[HF] P. Holy and S. Friedman, *Strong condensation*, unpublished.

[HS85] L. Harrington and S. Shelah, *Some exact equiconsistency results in set theory*, Notre Dame Journal of Formal Logic **26** (1985), no. 2, 178–188.

[Moo02] J. T. Moore, *Open colorings, the continuum, and the second uncountable cardinal*, Proceedings of the American Mathematical Society **130** (2002), 2753–2759.

[Nee08] I. Neeman, *Hierarchies of forcing axioms II*, Journal of Symbolic Logic **73** (2008), no. 2, 522–542.

[RS99] A. Rosłanowksi and S. Shelah, *Norms on possibilities I: forcing with trees and creatures*, Memoirs of the American Mathematical Society **141** (1999).

[Sch04] D. Schrittesser, Σ^1_3-*Absoluteness in forcing extensions*, Master's thesis, Universität Wien, 2004, Available at `http://www.logic.univie.ac.at/~david/papers/mthesis.pdf`.

[Sch07] D. Schrittesser, *Lightface* Σ^1_2-*indecribable cardinals*, Proceedings of the American Mathematical Society **135** (2007), 1213–1222.

[Sch10] D. Schrittesser, *Projective measure without projective Baire*, Ph.D. thesis, Universität Wien, 2010.

[She98] S. Shelah, *Proper and improper forcing*, Springer-Verlag, 1998.

[SS] D. Schrittesser and S. Shelah, ITP(ω_2) *is consistent with large continuum*, in preparation.

[VW] M. Viale and C. Weiß, *On the consistency strength of the proper forcing axiom*, preprint available at `http://fiesh.homeip.net/viale_weiss.pdf`.

[Wei] C. Weiß, *The combinatorial essence of supercompactness*, forthcoming.

[Wei10] ———, *Subtle and ineffable tree properties*, Ph.D. thesis, Universität München, 2010.

Diana Carolina Montoya Amaya
Universidad de Los Andes, Bogotá, Colombia
`dcmontoyaa [at] gmail.com`

Since I was studing the undergraduate program in Mathematics in the National University of Colombia I was interested on set theory, for that reason I worked on the final thesis work (which is a requirement to obtain the Mathematics degree) called *Definable Versions in Combinatorial Set Theory* which was supervised by professor Andres Villaveces. This work was based on the paper of Amir Leshem called On the Consistency of the Definable Tree Property on $\aleph_1$, I studied some simple forcing techniques and learnt some consistency results; also using the knowledge about weakly compact cardinals and Π^1_1- reflecting cardinals (introduced by Leshem), we studied the paper (preprint) *Weakly Compact Cardinals and κ- torsionless modules* written by Juan Nido, Pablo Mendoza and Luis Miguel Villegas; and proved a result about torsionless modules and κ- torsionless modules using the Π^1_1- reflecting cardinals . These work was awarded as the Best Undergraduate Thesis of the Mathematics Department and also was chosen to participate in the National Contest of final thesis works Otto de Greiff.

After, I started the Master Program in Mathematics in Andes University (Bogotá), I participate on the First Meeting of Set Theory between Venezuela and Colombia and also I was attending to a course on Forcing of the Reals teaching by professor Jorg Brendle from Kobe University (Japan), since that day I started to read some papers written by him; then he accepted to be my supervisor for the Master Thesis, called *Forcing Notions Presented as Quotients*, which was presented on June of the present year. In this work we studied mainly the results of Jindrich Zapletal and Michael Hrusak of the paper *Forcing with quotients* and answered an open question given by Bohuslav Balcar, Fernando Hernández-Hernández and Michael Hrušák in their paper *Combinatorics of Dense Subsets of the Rationals.*

Now, I'm starting the Phd program also in Andes University, and I want to keep doing research on set theory, also joint with professor Brendle we're going to prepare a paper to show the results of our work.

Dilip Raghavan
Fields Institute, Toronto, Canada
`raghavan [at] math.toronto.edu`

Most of my research has focused on the following areas: Tukey theory of ultrafilters on ω, maximal almost disjoint families of sets and functions, preservation theorems for iterated forcing, and consistency with CH.

Cofinal types of ultrafilters. Given ultrafilters $\mathcal{U}$ and $\mathcal{V}$ on ω, we say that $\mathcal{V}$ is *Tukey reducible to* $\mathcal{U}$, and write $\mathcal{V} \leq_T \mathcal{U}$ if there is a map $\phi : \mathcal{U} \to \mathcal{V}$ such that $\forall a, b \in \mathcal{U}\, [a \subset b \implies \phi(a) \subset \phi(b)]$ and $\forall e \in \mathcal{V} \exists a \in \mathcal{U}\, [\phi(a) \subset e]$. We say $\mathcal{U}$ and $\mathcal{V}$ are *Tukey equivalent*, and write $\mathcal{U} \equiv_T \mathcal{V}$, if $\mathcal{V} \leq_T \mathcal{U}$ and $\mathcal{U} \leq_T \mathcal{V}$. There is a general notion of Tukey reducibility for arbitrary directed posets, of which this is a special case. Several general structure and nonstructure theorems are known regarding Tukey types of uncountable directed sets. In the case of ultrafilters, Tukey reducibility is a coarser notion of reducibility than the well studied Rudin-Keisler (RK) reducibility. In joint work with Todorčević, we consider the question of when Tukey reducibility is equivalent to RK reducibility. This is similar in spirit to asking when a automorphism of $\mathcal{P}(\omega)/\mathrm{FIN}$ is induced by a permutation of ω, which was a famous problem in the history of set theory. The most outstanding question regarding cofinal types of ultrafilters is the following long standing problem of Isbell.

Question. *Is it consistent that for every ultrafilter $\mathcal{U}$ on ω, there exists $\{x_\alpha : \alpha < \mathfrak{c}\} \subset \mathcal{U}$ such that for every $A \in [\mathfrak{c}]^\omega$ $\left[\bigcap_{\alpha \in A} x_\alpha \notin \mathcal{U}\right]$?*

Almost disjoint families. We say that two infinite subsets a and b of ω are *almost disjoint or a.d.* if $a \cap b$ is finite. We say that a family $\mathscr{A}$ of infinite subsets of ω is *almost disjoint or a.d. in* $[\omega]^\omega$ if its members are pairwise almost disjoint. A *Maximal Almost Disjoint family, or MAD family in* $[\omega]^\omega$ is an infinite a.d. family in $[\omega]^\omega$ that is not properly contained in a larger a.d. family. Two functions f and g in ω^ω are said to be *almost disjoint or a.d.* if they agree in only finitely many places. We say that a family $\mathscr{A} \subset \omega^\omega$ is *a.d. in* ω^ω if its members are pairwise a.d., and we say that an a.d. family $\mathscr{A} \subset \omega^\omega$ is *MAD in* ω^ω if $\forall f \in \omega^\omega \exists h \in \mathscr{A}\, [|f \cap h| = \omega]$. We say that $p \subset \omega \times \omega$ is an *infinite partial function* if it is a function from some infinite subset $a \subset \omega$ to ω. An a.d. family $\mathscr{A} \subset \omega^\omega$ is said to be *Van Douwen* if for any infinite partial function p there is $h \in \mathscr{A}$ such that $|h \cap p| = \omega$. We answered an old question of Van Douwen by proving that Van Douwen families exist.

We have also answered a question of Shelah and Steprāns about almost disjoint families in $[\omega]^\omega$ that is closely related to the metrization problem for countable Fréchet groups. Let FIN denote the non-empty finite subsets of ω. Given an ideal $\mathcal{I}$ on ω, we say that $P \subset \mathrm{FIN}$ is *$\mathcal{I}$-positive* if $\forall a \in \mathcal{I} \exists s \in P\, [a \cap s = 0]$. Given an a.d. family $\mathscr{A} \subset [\omega]^\omega$, let $\mathcal{I}(\mathscr{A})$ denote the ideal on ω generated by $\mathscr{A}$. We say that an a.d. family $\mathscr{A} \subset [\omega]^\omega$ is *strongly separable* if for each $\mathcal{I}(\mathscr{A})$-positive $P \subset \mathrm{FIN}$, there is $a \in \mathscr{A}$ and $Q \in [P]^\omega$ such that

$\bigcup Q \subset a$. Thus this notion is gotten from the well known notion of a completely separable a.d. family by replacing integers with finite sets in the definition. Shelah has recently proved that completely separable a.d. families exist if $\mathfrak{c} < \aleph_\omega$. But we show that strong separability behaves differently by proving that it is consistent that there are no strongly separable a.d. families and $\mathfrak{c} = \aleph_2$.

The following are some interesting open problems regarding almost disjoint families.

Question. *Is there a completely separable a.d. family?*

Question. *Is there an uncountable a.d. family $\mathscr{A} \subset [\omega]^\omega$ such that for every $\mathcal{I}(\mathscr{A})$-positive $P \subset \mathrm{FIN}$, there is a $Q \in [P]^\omega$ consisting of pairwise disjoint sets so that $\forall a \in \mathcal{I}(\mathscr{A})\, [|a \cap (\bigcup Q)| < \omega]$?*

Question. *Is there an Sacks indestructible MAD family?*

Preservation theorems. We have answered a question of Kellner and Shelah by proving the following: Let γ be a limit ordinal and let $\langle \mathbb{P}_\alpha, \mathring{\mathbb{Q}}_\alpha : \alpha \leq \gamma \rangle$ be a countable support (CS) iteration. Suppose that for each $\alpha < \gamma$, $\Vdash_\alpha$ "$\mathring{\mathbb{Q}}_\alpha$ is proper" and that $\mathbb{P}_\alpha$ does not turn $\mathbf{V} \cap \omega^\omega$ into a meager set. Then $\mathbb{P}_\gamma$ does not do so either.

Question. *Let γ be a limit ordinal and let $\langle \mathbb{P}_\alpha, \mathring{\mathbb{Q}}_\alpha : \alpha \leq \gamma \rangle$ be a CS iteration. Suppose that for each $\alpha < \gamma$, $\Vdash_\alpha$ "$\mathring{\mathbb{Q}}_\alpha$ is proper" and $\mathbb{P}_\alpha$ does not add a Cohen real. Is it true that $\mathbb{P}_\gamma$ also does not add a Cohen real?*

Dominik Adolf
University of Münster, Germany
`dominik.adolf [at] uni-muenster.de`

I'm interested in a diverse array of set-theoretic subjects, including large cardinals, forcing and generic ultrapowers. I try to focus on inner model theory though.

Most of the last two years I spend on studying the core model induction. This is a techique used to compute lower bounds for consistency strength, applicable to a wide range of statements, say from pcf-theory (see [1]) or forcing axioms (see [2]).

The first step in any core model induction is the closure of the universe under $M_n^\#$ for every natural number n, i.e. there exists for every set X a mouse built over X and containing n woodin-cardinals bigger than the rank of X. To obtain this we utilize the K-existence dichotomy, which broadly states, that, assuming the closure of the universe under $M_n^\#$ for some n, either the universe is closed under $M_{n+1}^\#$ or for some set X the core model over X exists.

The next step is to show the mouse capturing condition W_α^* for all ordinals α. W_α^* states, that for any set of reals $U \in J_\alpha(\mathbb{R})$, such that both U and it's complement admit scales in $J_\alpha(\mathbb{R})$, any real x and any natural number n, there exists a mouse containing x and n woodins that "captures" U. It is an elementary fact, that W_α^* for all α yields $AD^{L(\mathbb{R})}$.

After this one can try to construct socalled HOD-mice, which are intrinsically linked to models of determinancy (see [4]).

At the moment though I'm writing on a paper together with Peter Koepke. The topic in question originates from my diploma thesis. We are looking for forcings, that function like Namba-forcing, but on cardinals bigger than $\aleph_2$. Say you have some big cardinal κ and some regular $\nu < \kappa$. Does there exist a forcing $\mathbb{P}$, which changes the cofinality of κ^+ to ν, without touching cardinals below κ^+? We managed to compute the consistency strength of this statement to be a measurable cardinal $> \kappa$, that has Mitchell-order at least η, where η is such that $\omega \cdot \eta = \nu$, if κ is regular. (The consistency strength for singular κ is one woodin cardinal.)

Furthermore I'm interested in any application of the stacking mice method introduced here: [3]

References

[1] M. Gitik, S. Shelah, R. Schindler, *Pcf theory and Woodin cardinals*, Lecture Notes in Logic **27** (2006) 172 – 205

[2] J. Steel, S. Zoble, *Determinancy from strong reflection*, preprint.

[3] R. Jensen, E. Schimmerling, R. Schinlder, J. Steel, *Stacking Mice*, Journal of Symbolic Logic **74** (2009) 315–335.
[4] G. Sargsyan, *A tale of hybrid mice*, PHD Thesis.

Edoardo Rivello
University of Turin, Italy
`edoardo.rivello [at] unito.it`

I am interested in any set-theoretical topic that origines its motivation from a philosophical, foundational or metamathematical issue.

I have just completed my Ph.D. with the thesis "Set Theory with a Truth Predicate", under the supervision of Alessandro Andretta (Turin university, Italy).

The Truth predicate for the universe of sets is employed in many areas of set theory, for instance to formulate some large cardinal axioms. But, by Tarski's Undefinability of Truth Theorem, the notion of Truth for the language of set theory (LST) is not formalizable in the same language [5].

Hence it is natural to ask for a formalization of Truth in an expanded language. There are, in general, two ways to do this. One is an Axiomatic Theory of Truth: you expand LST with a symbol for the satisfaction relation and you extend ZF adding axioms for this new symbol as, e.g., the Tarski's rules to handle connectives and quantifiers [3]; another way is to give a Truth-definition in an expansion of LST, e.g. by means of classes or by means of an elementary embedding of the universe in itself [1]. The semantical side of both approaches is the study of the so-called "full satisfaction classes", namely the interpretations of the satisfaction relation in models of ZF [4].

In my thesis I review some old and more recent results on Truth-axioms and Truth-definitions and give a sufficient condition for a transitive model of ZF to be a model of the Tarski's rules and of the Replacement-schema extended to the formulæ in which occurs the symbol for the satisfaction relation.

Now I am trying to link my technical background on Truth and set theory to the study of the Structural Reflection Principles recently proposed by J. Bagaria.

In the same time I am looking also to another area of set theory — Quine's *NF* (*New Foundation*) — and I am learning about the techniques employed in the current researches on the Consistency Problem for *NF* [2].

References

[1] P. Corazza: *The spectrum of elementary embeddings* $j : V \to V$, Ann. Pure Appl. Logic 139 (2006), pp. 327-399

[2] T.E. Forster: *AC fails in the natural analogues of L and the cumulative hierarchy that model the stratified fragment of ZF*, Contemporary Mathematics 36, 2004

[3] V. Halbach (ed. introd.): *Methods for investigating self-referential truth*, Studia Logica 68 n. 1 (2001), pp. 3-20

[4] H. Kotlarski: *Full Satisfaction Classes: a Survey*, NDJFL 32 n. 4, 1991

[5] A. Tarski: *The concept of truth in formalized languages* [in polish]. English tr. in [6]

[6] A. Tarski: *Logic, Semantics, Metamathematics. Papers from 1923 to 1938*, Oxford, Clarendon Press, 1956

Erin Carmody
City University of New York, USA
carmody.erin [at] gmail.com

As a student of Set Theory, I have been reading and doing exercises from Jech's *Set Theory* and Joel Hamkins' *Forcing and Large Cardinals.* While I do not yet have a research project, I have worked on a couple of detailed exercises. For one project I presented on the topic of Hausdorff Gaps as presented in Jech. Specifically, I presented the proof outlined in Theorem 29.7 - There exists an (ω_1, ω_1)-gap in ω^ω. Also, before the conference I will have presented on the construction of an $\aleph_2$-Aronszajn tree (which requires the assumption $2^{\aleph_0} = \aleph_1$). I am eager to continue to learn Set Theory, specifically forcing (which is the focus of my independent study this semester), and hope that this conference will give me a clearer picture of forcing and expose me to current areas of research in Set Theory.

Fabiana Castiblanco
University of São Paulo, Brazil
fcastq [at] ime.usp.br

I am a master math student at IME (Instituto de Matemática e Estatística) of University of São Paulo under supervision of Professor Artur Hideyuki Tomita. My interest lies in general and set-theoretic topology, particularly the unexpected behavior of topological spaces enriched with an algebraic structure when we consider ZFC with additional assumptions, v.g. CH or $\neg CH$, different forms of MA (Martin Axiom), like $MA_{countable}$, $MA_{\sigma-\text{centered}}$ or even the total failure of MA. Moreover, I am interested to study topological objects whose existence is independent from ZFC, results that are frequently proved using forcing and techniques which involve elementary substructures.

Currently, I am writing my master dissertation on existence of countably compact topological groups without non trivial convergent sequences using CH [Tka90], MA [vD80], selective ultrafilters [GFTW04, GT07] and forcing [KTW00, Tom03]; the construction of those groups allows to solve problems that apparently are not connected to their existence, such as the Wallace problem [RS96, Tom96], the non productivity of countably compactness in topological groups [vD80, Tom05a, Tom05b] and some questions related to independent group topologies [TY02].

References

[GFTW04] Salvador Garcia-Ferreira, Artur H. Tomita, and S. Watson. Countably compact groups from a selective ultrafilter. *Proc. Amer. Math. Soc.*, 133(3), 2004.

[GT07] Roberto E. Madariaga Garcia and Artur H. Tomita. Countably compact topological group topologies on a free Abelian groups from selective ultrafilters. *Topology Appl.*, 154(7):1470–1480, 2007.

[KTW00] Piotr B. Koszmider, Artur H. Tomita, and S. Watson. Forcing countably compact groups topologies on a larger free Abelian group. *Topology Proc.*, 25:563–574, 2000.

[RS96] Desmond Robbie and Sergey Svetlichny. An answer to A. D. Wallace's question about countably compact cancellative semigroups. *Proc. Amer. Math. Soc.*, 124(1), 1996.

[Tka90] M. G. Tkačenko. Countably compact and pseudocompact topologies on free Abelian groups. *Iz. VUZ.*, 34(5):68–75, 1990.

[Tom96] Artur H. Tomita. The Wallace problem: a counterexample from $MA_{\text{countable}}$ and p-compactness. *Canad. Math. Bull.*, 39(4):486–498, 1996.

[Tom98] Artur H. Tomita. The existence of initially ω_1 compact group topologies on free Abelian groups is independent of ZFC. *Comment. Math. Univ. Carolin.*, 39(2):401–413, 1998.

[Tom03] Artur H. Tomita. Two countably compact topological groups: One of size $\aleph_\omega$ and the other of weight $\aleph_\omega$ without non-trivial convergent sequences. *Proc. Amer. Math. Soc.*, 131(8):2617–2622, 2003.

[Tom05a] Artur H. Tomita. A solution to Comfort's question on the countable compactness of powers of a topological groups. *Fund. Math.*, 186:1–24, 2005.

[Tom05b] Artur H. Tomita. Square of countable compact groups without non-trivial convergents sequences. *Topology Appl.*, 153(1):107–122, 2005.

[TY02] M. G. Tkačenko and Ivan Yasčenko. Independent group topologies on Abelian groups. *Topology Appl.*, 122(1-2):425–451, 2002.

[vD80] Erick K. van Douwen. The product of two countably compact topological groups. *Trans. Amer. Math. Soc.*, 262:199–205, 1980.

Gido Scharfenberger-Fabian
Ernst-Moritz-Arndt Universität Greifswald, Germany
`gsf [at] uni-greifswald.de`

Set Theory. Combinatorial set theory and Boolean algebras, mainly Souslin tree constructions with special emphasize on the structural properties of the associated Souslin algebras. I constructed some Souslin algebras with strong homogeneity properties, such as *chain homogeneity*, i.e., there is only one order type of Souslin line associated to the algebra.

I also studied some forcing techniques applicable to Souslin trees.

Favourite open problem: *Is it consistent relative to ZFC, that there is exactly one Souslin line (without separable intervals, up to isomorphism)?*
I call this problem "Souslin's hypothesis minus one", as there would be essentially only one counter example to Souslin's hypothesis.

Proof Complexity. I recently studied the basics of proof complexity and am currently trying to get a grip on Krajíček's forcing style approach to proving lower bounds on the length of proofs of families of propositional tautologies. This problem is tightly bound to the famous NP vs. co-NP problem of computational complexity.

Giorgio Audrito
Turin University, Italy
giorgio.audrito [at] gmail.com

Definability and forcing extensions:

- Laver's Theorem and definability (with parameters in M) of a set-generic restriction M of V.
- Bukovsky's Theorem and characterization of V being a κ-cc (risp. size at most κ) generic extension of M.
- First-order formulations of the above two properties and Ground Axiom.

Approximation properties and consequences:

- κ-approximation in $H(\kappa)$.
- Properties of κ-approximated sets.
- Closure $\overline{M}^{\kappa}$ of a model M by κ-approximation.

Giorgio Laguzzi
Kurt Gödel Research Center Vienna, Austria
`giorgio.laguzzi [at] univie.ac.at`

My interests mainly concern the study of several regularity properties of the reals (and forcings associated with them): from the most common and oldest ones, like Lebesgue measurability (random forcing), Baire property (Cohen forcing), perfect set property and Ramsey property (Mathias forcing), to the most recent ones, like Miller measurabilty (Miller forcing), Laver measurability (Laver forcing) and Dominating measurability (Dominating forcing) (Sacks measurability in this sense cannot be actually consider a new property, since it coincides with the old Bernstein partition property). More precisely, I am interested in statements of the form

(1) *" every set of reals is regular* "*,

where *regular** represents any regularity property among those mentioned above. The basic model for all these properties is the Solovay model; however such a model is somehow too nice, since it satisfies all statements like (1), and furthermore an inaccessible cardinal is needful to get it. Hence, studies in this field may be divided into two main branches:

(a) the first one concerns the construction of models to separate statements of this type for different properties (i.e. models in which statement (1) holds for a certain regularity property but not for another one);
(b) the second one concerns to understand whether the existence of an inaccessible is really necessary to get a certain regularity property.

About (a), many examples of such models were introduced by Shelah, in particular to separate Lebesgue measurability, Baire property and perfect set property (one of these models is also due to a joint work of Di Prisco and Todorcevic). About (b), almost all cases has been solved during the years, except for two: Ramsey property and Laver measurability. One can notice that these two properties are strictly connected, since the Ramsey property may be seen as the uniform version of Laver measurability. Hence, many suggestive questions are still open in this area. In particular, in this period I am focusing the attention on this problem about the Laver measurability and I am rather confident that, like for the Baire property, one can construct the desired model without inaccessible cardinals.

Giorgio Venturi
Université Paris 7, France/ Scuola Normale Superiore Pisa, Italy
gio.venturi [at] gmail.com

My interests in set theory are both mathematical and philosophical. I'm attending my second year of PhD and since now I am focusing on the study of some consequences of the Forcing Axioms. In particular, I studied the problems related to Shelah's Conjecture (: every Aronszajn line contains a Countryman suborder) and the equivalent Five Element Basis Conjecture (: the orders $X, \omega_1, \omega_1^*, C$ and C^* form a five element basis for the uncountable linear orders any time X is a set of reals of cardinality $\aleph_1$ and C is a Countryman suborder). Moore showed that the Conjectures follow from PFA, but soon after has been discovered by König, Larson, Moore and Veličković that the consistency strength of the hypothesis can be reduced to that of a Mahlo cardinal, instead of that of PFA, whose upper bound is a supercompact cardinal and whose lower bound is a class of Woodin cardinals. Last year, Boban Veličković and I ([Veličković, Venturi 2010]), managed to give a more direct proof of the five element basis theorem, but still with the same hypotheses as in [König, Larson, Moore, Veličković 2008]

There are many problems related to this subject, that would be worth studying. First of all, a question that arise naturally is: do we really need some large cardinal strength for Shelah's Conjecture? Moreover it is interesting to see which are the influences of this Conjecture on the cardinality of the Continuum, because in the models of PFA, $2^{\aleph_0} = \aleph_2$, but if we do not need the consistency strength of PFA, can we find a model where Shelah's Conjectures holds, but the cardinality of the Continuum is difffers from $\aleph_2$?

Another subject I am interested in is the study of forcing of size $\aleph_1$. It has been proved, by Aspero and Mota, that PFA restricted to posets of size $\aleph_1$ is consistent with the continuum large. It would be interesting to see if it is possible to give a classification of this class of posets in the same way, under PFA, it is possible to classify the Aronszajn lines under the relation of be-embeddability.

References

[König, Larson, Moore, Veličković 2008] Bernhard König, Paul Larson, Justin Tatch Moore and Boban Veličković: *Bounding the consistency strength of a five element linear basis*, Israel Journal of Mathematics. 164(1): 1-18, 2008.

[Moore 2005] Justin Tatch Moore: *Structural analysis of Aronszajn trees*, Proceedings of the 2005 Logic Colloquium in Athens, Greece.

[Moore 2006] Justin Tatch Moore: *A five element basis for the uncountable linear orders*, Annals of Mathematics (2), 163(2): 669-688, 2006.

[Todorčević 2001] Stevo Todorčević : *Lipszhitz maps on trees*, Report 2000/01 number 13, Istitut Mittag-Leffler.

[Veličković, Venturi 2010] Boban Veličković, Giorgio Venturi: *A direct proof of the five element basis theorem.* Available on http://arxiv.org/abs/1012.0596, 2010.

Grigor Sargsyan
University of California, Los Angeles, USA
`grigor [at] math.ucla.edu`

Much of my research belongs to the general area of building canonical inner models for large cardinals and exploring the connections between inner model theory and descriptive set theory. I have been mainly occupied with proving the *Mouse Set Conjecture* (MSC), which is one of the central open problems of the two aforementioned areas of set theory. In my thesis, I developed the theory of hod mice which I used to prove some instances of MSC and applying the theory of hod mice to a more general setting with a goal of solving MSC is part of my future research plans. The main importance of MSC is that it can be used to obtain partial results on the *inner model problem*. The resolution of MSC will also increase the power of the *core model induction*, which is a very successful technique, due to Woodin, for evaluating lower bounds of the consistency strengths of various statements. I am also interested in other applications of the theory of hod mice. Examples of such applications are determining the consistency strengths of 1. the existence of *divergent* models of *AD* 2. the theory $AD_{\mathbb{R}} +$"θ is regular" 3. $\neg\Box_\kappa$ where κ is a singular strong limit cardinal and etc.

I have also worked on some questions of pure descriptive set theory that can be answered using techniques from inner model theory. One such question concerns the lengths of $\Game^k(\omega \times n - \Pi^1_1)$ prewellorderings. Outside descriptive set theory and inner model theory, I have worked in the area of large cardinals and forcing where I have been primarily working on problems surrounding the *identity crisis* phenomenon. My main contribution is a new way of forcing indestructibility for strong compactness that can be used to show the identity crisis type of results while main- taining some form of Laver indestructibility for the strongly compacts.

Hiroaki Minami
Kurt Gödel Research Center for Mathematical Logic, Vienna, Austria
minami [at] kurt.scitec.kobe-u.ac.jp

I'm interested in some structures on ω and cardinal invariants of the continuum related to these structure.

When we analyze the structure $(\omega)^\omega$ of infinite partitions of ω ordered by almost coarser $\sqsubset^*$, we can define cardinal invariants which is analogous to cardinal invariants on $([\omega]^\omega, \subset^*)$. We call the independence number for $(\omega)^\omega$ dual-independence number, denoted by $\mathfrak{i}_d$. As almost disjoint number $\mathfrak{a}$, we can show that if ZFC with measurable cardinal is consistent, then ZFC with $\mathfrak{u} < \mathfrak{i}_d$ is consistent. I conjecture that $\mathsf{cf}(\mathfrak{i}_d) = \omega$ is consistent as is $\mathsf{cf}(\mathfrak{a}) = \omega$.

Also I'm interested in mad families on ω, ideals on ω and relation among them. When we study those, Mathias-Prikry and Laver-Prikry type forcing are significant. Michael Hrušák and I prove that $\mathbb{M}(\mathcal{I}^*)$ adds a dominating real if and only if $\mathcal{I}^{<\omega}$ is P^+-ideal. Concerning to this results, it is known that $\mathfrak{b} = \mathfrak{c}$ implies that there exists a mad family such that $\mathbb{M}(\mathcal{I}(\mathcal{A})^*)$ adds a dominating real [1], where $\mathcal{I}(\mathcal{A})$ is ideal generated by $\mathcal{A}$. It is not known whether ZFC implies that there exists a mad family $\mathcal{A}$ such that $\mathbb{M}(\mathcal{I}(\mathcal{A})^*)$ adds a dominating real.

For ultrafilter, it is known that $\mathfrak{d} = \mathfrak{c}$ implies that there exists an ultrafilter $\mathcal{U}$ such that $\mathbb{M}(\mathcal{U})$ doesn't add dominating real [2]. I'm trying to know when we can construct such a mad family and such an ultrafilter by using our characterization.

References

[1] Jörg Brendle, *Mob families and mad families*, Archive for Mathematical Logic, 37, (1998), 183–197.
[2] R. Michael Canjar, *Mathias forcing which does not add dominating reals*, 10r, No 4, December (1998), 1239–1248.
[3] Michael Hrušák and Hiroaki Minami, *Mathias forcing and Laver forcing associated with ideals*, preprint.

Hossein Lamei Ramandi
University of Tehran, Iran
hlramandi [at] yahoo.com

I am interested in infinitary combinatorics and forcing, and their applications to Topology and Analysis. Below I describe what I am studying now.

In [2] N. Weaver discussed some set theoretic problems about the *Calkin algebra* and one of them was Hadwin's conjecture. In [1] D. Hadwin showed that under CH all maximal chains in $\mathcal{P}$ – the lattice of projections in the Calkin algebra, $\mathcal{C}$ – are order isomorphic, and conjectured that it is equivalent to CH. In [3] E. Wofsey introduces an analogy between $\mathcal{C}$, and, $\mathrm{P}(\omega)/fin$ and shows that " there exists non-isomorphic maximal chains in $\mathcal{P}$ " is consistent with ZFC.

At the moment I am a master student at University of Tehran . In my master thesis I will go through the theorems in [3] and I will try to work on Hadwin's conjecture.

References

[1] D. Hadwin, maximal nests in the Calkin algebra, Proc. Amer. Math. soc. **126** (1998), 1109–1113

[2] N. Weaver, Set Theory and C^*-algebras, Bull. Symb. Logic **13** (2007), 1–20

[3] E. Wofsey $\mathrm{P}(\omega)/fin$ and projections in the Calkin algebra, Proc. Amer. Math. Soc. **136** (2008), no. 2, 719–726

Ioanna Matilde Dimitríou

Rheinische Friedrich-Wilhelms-Universität Bonn, Germany

`dimitri [at] math.uni-bnn.de`

I am a PhD student at the Logic group of Bonn, under the supervision of Peter Koepke. My main area of interest is set theory ZF under the *negation of the Axiom of Choice* ($\neg$AC). In particular, I'm working on large cardinals, singular cardinal patterns, and variants of the Chang conjecture, all under $\neg$AC. The main method I am using is symmetric forcing.

I finished my Masters degree in the Institute for Logic, Language, and Computation, at the University of Amsterdam, the Netherlands. My masters thesis was supervised by Benedikt Löwe and it's entitled "Strong limits and inaccessibility with non-wellorderable powersets". Work from this was published in a joint paper with Andreas Blass and Benedikt Löwe, [BDL06].

My PhD thesis is now at the last stages. The first main chapter is on symmetric forcing and the approximation lemma. There I present several models of ZF+$\neg$AC and large cardinals made small (successor cardinals). Symmetric class forcing is also discussed there. There is also a section on second order arithmetic (SOA), in particular a model of SOA in which all sets of reals are Lebesgue measurable, have the Baire property and the perfect set property. This model is constructed from just a model of ZFC by collapsing all ordinals to ω and it's part of a joint paper with Peter Koepke and Michael Möllerfeld, [DKM], which is under preparation.

The next chapter is on patterns of singular cardinals of cardinality ω that is based mainly on Moti Gitik's paper "All uncountable cardinals can be singular" [Gi80]. From this Chapter there is a joint paper with Arthur Apter and Peter Koepke, entitled "The first measurable cardinal can be the first uncountable regular cardinal at any successor height" [ADK].

I am currently finishing the last chapter which is on the variants of the Chang conjecture, in which I also use some arguments and black boxes from core model theory. There, variants of the Chang conjecture that are very strong under AC, are shown to be equiconsistent (under $\neg$AC) with very weak hypotheses (e.g., an Erdős cardinal). The core model arguments there are based mainly on [DJK79] and on [DK83].

References

[Gi80] Moti Gitik. All uncountable cardinals can be singular, *Israel Journal of Mathematics* 35:61–88, 1980

[ADK] Arthur Apter, Ioanna Dimitríou, and Peter Koepke. The first measurable cardinal can be the first uncountable regular cardinal at any successor height. *Submitted at Mathematical Logic Quarterly*

[DKM] Ioanna Dimitríou, Peter Koepke, and Michael Möllerfeld. Topological regularities in second order arithemetic. *Under preparation*

[BDL06] Andreas Blass, Ioanna Dimitríou, and Benedikt Löwe. Inaccessible cardinals without the axiom of choice. *Fundamenta Mathematicae* 194:179–189, 2006

[DJK79] Hans D. Donder, Ronald B. Jensen, and Bernd Koppelberg. Some applications of the core model. In Ronald B. Jensen, editor, *Set theory and model theory*, volume 872 of *Lecture Notes in Mathematics*, pages 55–97. Springer, 1979.

[DK83] Hans D. Donder and Peter Koepke. On the consistency strength of "accessible" Jónsson cardinals and of the weak Chang conjecture. *Annals of Pure and Applied Logic*, 25:233–261, 1983.

Joel David Hamkins
City University of New York, USA
jhamkins [at] gc.cuny.edu

I conduct research broadly in the area of mathematical logic, with a focus on set theory and with particular attention to the mathematics and philosophy of the infinite.

A principal concern has been the interaction of forcing and large cardinals, two central concepts in set theory. The general theme is the question: How are large cardinals affected by forcing? I have investigated the indestructibility phenomenon of large cardinals, introducing the lottery preparation in order to do so, as well as the lifting property, occurring when all the large cardinal embeddings of an extension are lifts of embeddings definable in the ground model.

I have introduced several new forcing axioms, which are expressed by a fundamental interaction of forcing and truth, rather than by a combinatorial property involving dense sets. The Maximality Principle, for example, asserts that any statement that is forceable in such a way that it remains true in all further forcing extensions is already true. Considerations of parameters make the axiom range in strength from ZFC up through the large cardinal hierarchy.

In related work, I have introduced the modal logic of forcing, where a statement of set theory is possible if it holds in some forcing extension and necessary if it holds in all forcing extensions. Together with Löwe, I proved that the ZFC-provably valid modal principles of forcing are exactly the assertions of the modal theory S4.2.

Together with Reitz and Fuchs, I have introduced the topic of set-theoretic geology, focused on how the set-theoretic universe relates to its various ground models and those of its forcing extensions. The mantle, for example, is the intersection of all grounds, and it turns out that every model of ZFC is the mantle of another model of ZFC.

I have worked in group theory and its interaction with set theory in the automorphism tower problem and in computability theory, particularly the infinitary theory of infinite time Turing machines, which I introduced with Kidder and Lewis. Engaging with the emerging subject known as the philosophy of set theory, I have introduced and defended a multiverse perspective.

Jonathan L. Verner
Charles University, Prague, Czech Republic
`jonathan.verner [at] matfyz.cz`

My current research focuses on questions concerning ultrafilters. Recently, new connections have been discovered between strong P-points and Mathias forcing and several interesting questions remain open, e.g.

Question Is is the relativized ultrafilter-Mathias forcing almost ${}^{\omega}\omega$-bounding provided the ultrafilter is a strong P-point?

Question Is the iteration of the natural forcing to add a strong P-point followed by killing this ultrafilter via Mathias forcing Π^1_1 on Σ^1_1?

Question Is being a Canjar filter equivalent to being a strong P-filter for meager-filters?

Of course, there are also other very intersting questions, which are probably much harder:

Question [1] Is it provable in ZFC that there is a non-meager P-filter?

One can also look at ultrafilters from a topological perspective and find interesting questions. For example there are points in ω^* which are limit points of a countable set without isolated points, however the neighborhood traces on any countable subset generate an ultrafilter. Put in a different way, these points cannot be $\mathcal{F}$-limits of a countable sequence for any filter which is not an ultrafilter. It is a standard fact that no point in ω^* can be a $\mathcal{FR}$-limit of a sequence. Recently I learned from T. Banach, that there are points, which can be $\mathcal{F}$-limits of some sequence with $\mathcal{F}$ a meager filter. This cannot happen if $\mathcal{F}$ is an F_σ-filter or an analytic P-filter.

Question Is there an analytic filter $\mathcal{F}$ such that the $\mathcal{F}$-limit of some sequence in ω^* exists?

References

[1] Kanamori, A. *Some combinatorics involving ultrafilters*, Fund. Math., vol 100, 1978, pp.145–155.

Juris Steprans
York University, Toronto, Canada
steprans [at] yorku.ca

My chief interest is in the applications of set theoretic methods, mostly cardinal invariants and forcing arguments, to questions in analysis. Examples of such questions include the geometric measure structure of Euclidean spaces, cardinal invariants of topological groups and, recently, the Gharamani-Lau Conjecture on the topological centre of the Arens product of the measure algebra.

Katie Thompson
Vienna University of Technology, Austria
thompson [at] logic.univie.ac.at

My research centres around the classification of relational structures (e.g. orders, graphs) and those with extra topological structure (e.g. ordered spaces, Boolean algebras) via the embeddability relation. An embedding is generally an injective structure-preserving map between structures. For example, for linear orders, the ordering is preserved in embeddings. The embeddability relation, $A \leq B$ iff A can be embedded into B, is a quasi-ordering of the structures. I study many aspects of this quasi-order: the top (universal structures or families), the bottom (prime models and bases) and internal properties of the embeddability structure (chains and antichains, the bounding number). Results in these areas are often independent of

ZFC and require additional cardinal arithmetic assumptions, combinatorial principles (e.g. diamond, club guessing), forcing techniques (as exotic as oracle-proper) or forcing axioms (e.g. PFA) to decide them.

In studying these questions, I developed a technique together with S. Friedman known as the "tuning fork method". This is a way of using uncountable versions of Sacks forcing ([4]) to change some properties (e.g. cardinal arithmetic) at measurable cardinals while preserving the measurability. Previously this had been done with Cohen forcing by Woodin and Gitik ([3]); our technique is not only simpler but also leads to additional applications. Friedman and I for instance combined this method with Prikry forcing to give a model in which there is no universal graph at the successor of a singular cardinal. The tuning fork method has also been used by Friedman and Magidor ([2]) to control the number of normal measures at a measurable cardinal. J. Cummings and I are working on extending these results to control configurations of measures in the Mitchell order.

My long-term projects are to look into models of set theory where GCH fails everywhere ([1]) and also models where the continuum is at least $\aleph_3$.

For a list of my papers, see

http://www.logic.univie.ac.at/∼thompson

References

1. M. Foreman and W.H. Woodin, *The generalized continuum hypothesis can fail everywhere*, Annals of Mathematics **133** (1991), no. 1, 1–35.
2. S. Friedman and M. Magidor, *The number of normal measures*, Journal of Symbolic Logic **74** (2009), no. 3, 1069–1080.
3. M. Gitik, *The negation of the singular cardinal hypothesis from* $o(\kappa) = \kappa^{++}$, Annals of Pure and Applied Logic **43** (1989), no. 3, 209–234.
4. A. Kanamori, *Perfect-set forcing for uncountable cardinals*, Annals of Mathematical Logic **19** (1980), 97–114.

Kevin Fournier
University of Lausanne, Switzerland
Kevin.Fournier [at] unil.ch

I'm a first year PhD student descriptive set theory, and my research centers around reductions by continuous function and Wadge hierarchy on the topological space Λ^ω, where Λ is a non-empty set. Given the work done by my supervisor Jacques Duparc for the borel sets, I try to generalize some interesting results to other topological classes with appropriate closure and determinacy, in order to get a very fine description of them. In particular, I'm studying Δ^1_2 sets under projective determinacy.

Kostas Tsaprounis
University of Barcelona, Spain
kostas.tsap [at] gmail.com

I am mainly interested in the various hierarchies of $C^{(n)}$-cardinals, as introduced in [1]. Recall that $C^{(n)}$ is the closed unbounded proper class of ordinals that are Σ_n-correct in the universe i.e. $C^{(n)} = \{\alpha : V_\alpha \prec_n V\}$, for $n \in \omega$. Now, given an elementary embedding $j : V \longrightarrow M$ (with critical point κ and M transitive) associated to any of the standard large cardinal notions, we may ask whether $j(\kappa) \in C^{(n)}$ holds (for any $n \in \omega$).

This question gives rise to the $C^{(n)}$-version of the large cardinal notion at hand, by modifying the usual elementary embedding definition so as to require, in addition, that $j(\kappa) \in C^{(n)}$. Consequently, we get (apparently) new large cardinal hierarchies such as $C^{(n)}$-measurables, $C^{(n)}$-(super)strongs, $C^{(n)}$-supercompacts etc. Various results about these hierarchies have highlighted their strong reflectional nature. Still, there are many unsolved questions even at the lowest levels e.g. regarding $C^{(1)}$-supercompacts. I am currently working on the latter and some related issues.

1. Bagaria, J., $C^{(n)}$ - *cardinals* . Submitted.

Lauri Tuomi
University of Paris 7, France
tuomi [at] logique.jussieu.fr

I am a PhD student at Université Paris 7. At the moment I am mostly interested in questions about absoluteness of the following form: Suppose that $V \subseteq W$ are models of set theory with the same cardinal numbers and $V \models \exists x \phi(x)$. Can we find an object $A \in V$ satisfying $\phi(A)$ in both V and W?

In particular, I am interested in the question above in the case that κ is an uncountable cardinal and $\phi(A)$ states that A is a partition of κ into stationary sets. For example from [2] we know that ω_1 can be divided into $\aleph_0$ many stationary sets in V, all of which remain stationary in W. On the other hand, by [1], there is a forcing extension $V[G]$ of V which preserves ω_1 but no partition of ω_1 into $\aleph_1$ many stationary sets remains such in $V[G]$ (although greater cardinals might be collapsed.) To what extent can these results be generalized for $\kappa > \omega_1$?

These kind of questions are of particular interest when V and W model forcing axioms. For example, by results in [4], if both $V \subseteq W$ model PFA, and for every κ, the ω-cofinal ordinals below κ^+ can be partitioned into κ many stationary sets in a way described above, then $\mathrm{Ord}^\omega \cap V = \mathrm{Ord}^\omega \cap W$. It has been further conjectured in [3] that if $V \subseteq W$ both satisfy PFA, then $\mathrm{Ord}^{\omega_1} \cap V = \mathrm{Ord}^{\omega_1} \cap W$. The basic informal question is: to what extent does PFA (or MM) fix its models?

1. Paul B. Larson: The nonstationary ideal in the $\mathbb{P}_{max}$ extension, The Journal of Symbolic Logic Volume 72, Number 1, 2007

2. Stevo Todorčević: Partitioning pairs of countable ordinals, Acta Mathematica, vol. 159 (1987), no. 3-4, pp. 261-294
3. Boban Veličković: Logic Colloquium 2006, Lect. Notes in Logic, Association for Symbolic Logic, Chicago, IL, (2009), pp. 328–360.
4. Matteo Viale: The Proper Forcing Axiom and the Singular Cardinal Hypothesis, Journal of Symbolic Logic 71(2) (2006) 473-479

Liuzhen Wu
Kurt Gödel Research Center Vienna, Austria
`liu.zhen.wu [at] univie.ac.at`

My research mainly focus on strong condensation principle(SC) and precipitous ideal on ω_1. SC was introduced by Woodin in [3], which is an abstract version of Condensation Lemma in L. Independently S. Friedman [1] and I [4] construct set-size forcings for SC on $H(\omega_2)$. On the other hand, in [2] Schimmering and Velickovic show that SC on $H(\omega_3)$ refutes the existence of precipitous ideal on ω_1. Now I am looking for a set forcing for SC on $H(\omega_3)$ or some fragment of it which may provide a solution to the "Lager Cardinal entails precipitous ideal" problem.

Also, I am interesting in the relation between canonical complete ideals and nonstationary ideals. There are forcing notions with respect to some canonical ideals on ω_1 (e.g club guessing ideal, $\diamond$ ideal etc.) which force these ideals to be $NS \restriction S$ for some stationary $S \subset \omega_1$. The further step is to seek for the consistency strength of the statement that all normal, countably complete ideal on ω_1 is of the form $NS \restriction S$ for some stationary $S \subset \omega_1$. This statement is a consequence of the saturation of NS_{ω_1}, hence the strength is below one Woodin cardinal.

1. Sy. Friedman, Forcing Condensation, preprint
2. Ernest Schimmerling, Boban Velickovic. Collapsing Function. Mathematical Logic Quarterly 50 (2004), No 1, 3-8

3. W. Hugh Woodin, The Axiom of Determinancy, Forcing Axioms, and the Nonstationary Ideal, de Gruyter Series in Logic and its Applications 1, Walter de Gruyter & Co., Berlin, 1999
4. Liuzhen Wu. Forcing Strong Condensation over $H(\omega_2)$ and Precipitous Ideal. PhD thesis.

Luís Pereira
University of Lisbon, Portugal
`pereira [at] logique.jussieu.fr`

My area of research is the Combinatorics of Singular Cardinals with an emphasis on Cardinal Arithmetic. I am insterested in connections between the PCF conjecture and more standard combinatorics. During this workshop I would like to solve, or acquire the tools necessary to solve, the following two questions which are related to the PCF conjecture ([3]):

1) Prove that in L there are continuous tree-like scales in the product of inaccessible cardinals.

This has already been done for the product of successor cardinals ([2]) in the 80's. This problem might involve notions from the construction of morasses in $L[U]$.

2) Translate a part of 1) into extender ultrapowers as in [1] and give a direct proof that "$0^\sharp$ exists" follows from the unary version of Shelah's Approachable Free Subset Property ([3]).

The purpose of this is to build the framework to then go to higher large cardinals, hopefully up to a sharp for a strong cardinal.

1. J. Cummings, M. Foreman, E. Schimmerling, *Organic and Tight*, Annals of Pure and Applied Logic 160 (2009), 22-32
2. H. D. Donder, R. B. Jensen, and L. J. Stanley, *Condensation-coherent global square systems*, In A. Nerode and R.A. Shore, editors, *Recursion Theory*, volume 42 of Proc. of Symp. in Pure Math, pages 237-258, Providence 1985, AMS
3. L. Pereira, *The PCF conjecture and large cardinals*, J. Symbolic Logic 73 (2008), 674-688

Luz María García Ávila
University of Barcelona, Spain
luzsabinaw [at] gmail.com

Most of the classical combinatorial results of Ramsey type can be proved using Generic Absoluteness arguments. The traditional proofs of classical results, e.g., the Galvin-Prikry or Silver's theorems, hinge on a careful analysis of Borel or analytic partitions. But if one wants to generalize these results to more complex partitions, then one not only needs to assume some extra set-theoretic hypothesis – such as large cardinals, determinacy, or forcing axioms –, but the proof itself needs to be adapted accordingly. The advantage of using generic absoluteness is that the same proof for the Borel case generalizes readily to more complex partitions, under the additional hypothesis that the universe is sufficiently absolute with respect to its forcing extensions by some suitable forcing notions. In the case of the Ramsey property for sets of reals, the associated forcing notion is Mathias' forcing. In the case of perfect set properties, such as the Bernstein property, the associated forcing notions are Sacks forcing and its Amoeba. Typically, to each kind of partition property there are associated two forcing notions: P and Amoeba-P, so that assuming a sufficient degree of generic absoluteness under forcing with them, one can prove the desired Ramsey-type results. The combinatorial core of the problem turns out to be the following: prove that every element of the generic object added by Amoeba-P is P-generic.

Marcin Sabok
University of Illinois at Urbana-Champaign, USA
msabok [at] gmail.com

I am especially interested in the theory of *idealized forcing.* This is a topological and descriptive analysis of forcing notions of the form P_I =Bor$(X)/I$, where X is a Polish space and I is a σ-ideal on it. There are two main directions in which the theory has made major progress. The first one develops forcing techniques that are used to force various behaviours of cardinal invariants and other phenomena on the real line. And the second one which applies forcing and absoluteness techniques to obtain new results in descriptive set theory.

One of the recent developments in the second direction (my joint work with Jindra Zapletal [2]) gives a new, purely descriptive set-theoretical dichotomy, which essentially uses idealized forcing. The result says that among σ-ideals generated by closed sets there are only two, drastically different cases which can occur. We say that a σ-ideal I has the *1-1 or constant* property if every Borel function defined on a Borel I-positive set can be restricted to a Borel I-positive subset, on which it is either 1-1 or constant. This is equivalent to saying that the forcing adds a minimal real degree. Of course, if P_I adds a Cohen real, then any name for it can be translated to a Borel function which cannot be restricted to be either 1-1 or constant. Now, the result says that a Cohen real is the only obstacle: for any σ-ideal I generated by closed sets, either P_I adds a Cohen real, or else I has the 1-1 or constant property. In many cases it is relatively easy to exclude the Cohen real (ω^ω-bounding, Laver property, etc.) and the 1-1 or constant property is a strong and useful statement. It can be also treated as a canonization result for smooth equivalence relations, i.e. saying that such equivalences trivialize after restriction to a Borel I-positive set. Now, there is a natural question for which σ-ideals and which equivalences the canonization can be obtained. This is work in progress, joint with Vladimir Kanovei and Jindra Zapletal.

Another interesting development in descriptive set theory, motivated by idealized forcing, is a recent result about the complexity of Ramsey-null sets. More precisely: the complexity of codes for analytic Ramsey-null sets. In many arguments in idealized forcing (esspecially about the iteration) it is important that the σ-ideal is absolutely definable. In most cases the σ-ideal is definable by a $\mathbf{\Pi}^1_1$ (or at least $\mathbf{\Delta}^1_2$) formula, which is absolute for countable models. A question that was first raised in idealized forcing (it was also asked independently by Daisuke Ikegami), is whether the σ-ideal of the Mathias forcing is definable in such way. In a recent result [1], I showed that it is not, and the reason is interesting from the descriptive set-theoretical point of view. It turns out, that the set of codes for Ramsey-positive analytic sets is $\mathbf{\Sigma}^1_2$-complete and this is a surprising analogon of the same phenomenon on the lower level of the projective hierarchy: the set of codes for uncountable (i.e. Sacks-positive) analytic sets is $\mathbf{\Pi}^1_1$-complete, which is an old and classical theorem of Hurewicz.

1. Sabok M., *Complexity of Ramsey-positive sets*, submitted,
2. Sabok M., Zapletal J.,*Forcing properties of ideals of closed sets*, Journal of Symbolic Logic, to appear.

Marek Wyszkowski
University of Kiel, Germany
`mafkonaut [at] googlemail.com`

I am interested in forcing, large cardinals and descriptive set theory. My research interest at the moment is the *splitting tree forcing*, a tree forcing that was found to characterize the countably splitting analytic subsets of the reals by Otmar Spinas in [1]. For a tree forcing S one can define an ideal of small subsets of the reals by $I(S) := \{X \subseteq 2^\omega | \; \forall p \in S \; \exists q \leq p : [p] \cap X = \emptyset\}$. Several consistency results regarding the cardinal invariants of these ideals and their relationships are known for a wide array of tree forcings, for example the consistency of $MA + \neg CH + Cov(I(S)) = \omega_1$, where S is the

Sacks forcing (see [2]). I am using iterated tree forcing constructions to find analogous results for the splitting tree forcing that are left open. Later research interests will be more general questions regarding splitting reals, for example to find a similar characterization for splitting hereditary subsets of reals of Borel complexity.

1. Otmar Spinas *Analytic countably splitting families*, Journal of Symbolic Logic **69** (2004), pp.101-117.
2. H. Judah, A. Miller, S. Shelah *Sacks Forcing, Laver Forcing and Martin's Axiom*, Arch. Math. Logic **31** (1992), no.3, pp.145-161.

Marios Koulakis
University of Münster, Germany
mkoul_01 [at] uni-muenster.de

My research interests lie in inner model theory and coding techniques (such as Jensen's coding theorem). Specifically, I am interested in how the methods from those fields can be combined in different applications.

An example is the following theorem by J. Steel, which provides a negative answer to the 12th Delfino problem. The 12th Delfino problem asks whether the following statement holds true:

$$ZFC + \triangle \vdash PD$$

where
$\triangle$ = every projective set is Lebesque measurable, has the Baire property and can be projectively uniformized.

Steel proved that the consistency strength of $\triangle$ is strictly less than a $Woodin$ cardinal thus the above can not hold. The proof involves the use of universally Baire sets and premice of a specific kind that are used in defining the projective uniformization functions. Coding comes into play, in the part where by collapsing certain cardinals, trees that represent universal $\mathbf{\Pi}^1_n$ sets are created.

My plans are to try to extend those results and investigate the possible types of coding into core models.

Matteo Viale
University of Torino, Italy
matteo.viale [at] unito.it

On the notion of guessing model. My current research focuses on the notion of guessing model. This has been analyzed and introduced in [1]. The ultimate and most likely out of reach ambition in this work is to provide by means of guessing models useful tools to show that for a given model W of MM, $(\aleph_2)^W$ has an arbitrarily high degree of supercompactness in some simply definable inner model V.

A guessing model come in pair with an infinite cardinal δ:

- $\aleph_0$-guessing models provide an interesting characterization of all large cardinal axioms which can be described in terms of elementary embedding $j : V_\gamma \to V_\lambda$. In particular supercompactness, hugeness , and the axioms I_1 and I_3 can be characterized in terms of the existence of appropriate $\aleph_0$-guessing models.
- In a paper with Weiss [2] we showed that PFA implies that there are $\aleph_1$-guessing models, and that in many interesting models W of PFA such $\aleph_1$-guessing models M can be used to show that in some inner model V of W, $M \cap V$ is an $\aleph_0$-guessing models belonging to V and witnessing that $\aleph_2$ is supercompact in V.
- In [1] I also outline some interesting properties guessing models have in models of MM. For example assume θ is inaccessible in W, then:
  (1) If W models PFA, then for a stationary set G of $\aleph_1$-guessing models $M \prec H_\theta$ the isomorphism-type of M is uniquely determined by the ordinal $M \cap \aleph_2$ and the order

type of $M \cap Card$ where $Card$ is the set of cardinals in H_θ.

(2) In the seminal paper of Foreman Magidor and Shelah [4] on Martin's maximum and in a recent work by Sean Cox [3] several strong forms of diagonal reflections are obtained, for example Cox shows:

> Assume MM holds in V. Then for every regular θ there is S stationary set of models $M \prec H_\theta$ such that every $M \in T$ computes correctly stationarity in the following sense: *For every $X \in M$ and every set $R \in M$ subset of $[X]^{\aleph_0}$ if R is projectively stationary in V then R reflects on $[M \cap X]^{\aleph_0}$.*

(3) We can improve (1) and (2) above to further argue that in a model V of MM, $G \cap S$ is stationary.

Such results even if rather technical are attributing to $\aleph_2$ properties shared by supercompact cardinals in the sense that $\aleph_0$-guessing models M are characterized by property (1) when $\aleph_2$ is replaced by some suitable inaccessible cardinal $\kappa \in M$ and satisfy many strenghtenings of property (2).

1. Matteo Viale, *On the notion of guessing model*, submitted, available at: `http://fiesh.homeip.net/guessing_model.pdf`, 2010, 17 pages.
2. Matteo Viale, Christoph Weiss, *On the consistency strength of the proper forcing axiom*, submitted, available at: `http://fiesh.homeip.net/viale_weiss.pdf`, 2010, 20 pages.
3. Sean Cox, *The diagonal reflection principle*, submitted, available at: `http://wwwmath.uni-muenster.de/logik/Personen/Cox/Research/DRP.pdf`, 2010, 11 pages.
4. Matthew Foreman, Menachem Magidor, Saharon Shelah *Martin's maximum, saturated ideals, and nonregular ultrafilters. I*, Annals of Mathematics. Second Series, **127(1)** (1988), 1–47.

Mayra Montalvo Ballesteros
University of Leeds, UK
ariesmayra [at] gmail.com

I have been working on constructions of rigid structures. A structure $\mathcal{A}$ is said to be *rigid* if its only automorphism is the identity. For instance, any finite linear ordering, or indeed any ordinal, is rigid. I also consider wider classes of maps on $\mathcal{A}$ than automorphisms, viz. embeddings, endomorphisms, epimorphisms, monomorphisms, bimorphisms (which are all various weakenings of the notion of automorphism). I have mainly considered dense linear orders without endpoints (though graphs are another case of interest) and given a number of constructions to illustrate what can happen.

A classical construction due to Dushnik and Miller [2] shows (using the Axiom of Choice) that $\mathbb{R}$ has a dense subset X of cardinality $2^{\aleph_0}$ which is rigid with respect to automorphisms. It is easy to modify this example to make it also rigid with respect to embeddings and epimorphisms (the latter are maps $f : X \to X$ which is surjective and such that $x \leq y \Rightarrow f(x) \leq f(y)$). In [1] is shown how to find such an X which is rigid for automorphisms but which admits *many* embeddings. A more precise question is to enquire what the possible *values* are of the embedding monoid $Emb(X, \leq)$ for an automorphism rigid $(X, \leq)$. For instance I can show that is can be isomorphic to $(\mathbb{N}, +)$, with a similar result for the epimorphism monoid $Epi(X, \leq)$.

For chains, only four of the possible six monoids are distinct, namely $Aut(X, \leq), Emb(X, \leq), Epi(X, \leq)$ and $End(X, \leq)$, and I can demonstrate all possible consistent combinations of equalities and inequalities between these. Work on the questions about what the moniods can actually be is still in progress.

1. Droste, M., Truss, J. K. *Rigid chains admitting many embeddings.* Proc. Amer. Math. Soc. **129** (2001), pp. 1601-1608.
2. Duchnik, Ben and Miller, E. W., *Concerning similarity transformations of linearly ordered sets.* Bull. Amer. Math. Soc, **46** (1940), pp. 322-326.

Michal Korch
Warsaw University, Poland
m_korch [at] mimuw.edu.pl

My interest in mathematics concerns compact scattered spaces obtained by the methods of the infinite combinatorics and their applications in Banach space theory. By scattered space I mean a space in which every non-empty subset has an isolated point. I am interested only in such compact and Haussdorff, thus normal, spaces. Some examples of such spaces are: Ψ-spaces, the Kunen space and the Ciesielski-Pol space.

It is easy to see that every compact scattered space is also zero-dimensional. Therefore every such space corresponds to an Boolean algebra by the Stone duality. It turns out [6] that the class of Boolean algebras obtained this way is exactly the class of superatomic algebras defined earlier by Mostowski and Tarski [2], that is algebras in which every non-zero element in every subalgebra has an atom below it.

Metrizable compact scattered spaces have clear classification due to Mazurkiewicz and Sierpinski [1], which says that every compact scattered metrizable space is homeomorphic to a countable successor ordinal with ordinal topology. Therefore every countable superatomic algebra is isomorphic to the algebra of clopen sets of some countable successor ordinal. But uncountable superatomic algebras make a much larger class and classification of that class is still beyond the scope of mathematical research.

Nevertheless one can define some important parameters of an superatomic algebra, such as height or width of an algebra [6]. Intuitively the height parameter measures when the process of taking iterated Cantor-Bendixon derivative of a the Stone space dual to the algebra will stabilize (it will stabilize on $\varnothing$ if and only if the space was scattered) and the width determines maximum cardinality of the set of isolated points in the spaces obtained in this process.

There are some open problems concerning these parameters. For example it is not known if there exists an superatomic Boolean algebra of countable width and height ω_3. Also there is not much research concerning the continuous maps between compact scattered spaces. For example the question whether exists (under ZFC) a compact scattered space K of uncountable height and countable width without non-trivial continuous maps, where by trivial map I mean a map $f\colon K \to K$, for which exists a countable subset $A \subseteq K$, such that f restricted to $K \setminus f^{-1}[A]$ is identity, is also open.

We can also consider Banach spaces of real-valued continuous functions on a compact scattered space. It is well known [7] that the dual space of $C(K)$, where K is compact and Haussdorff, is the space $M(K)$ of signed Radon measures on K. But when K is scattered every $\mu \in M(K)$ is of form $\mu(x) = \sum_{n=1}^{\infty} a_n x(t_n)$ for some $t_1, \ldots, t_n \in K$ and $\sum |a_n| < \infty$ – see [4].

Banach spaces of form $C(K)$, where K is compact and scattered, play also significant role in the Banach space theory as examples and counterexamples. For example in 1930 J. Schreier proving that dual space of $C(\omega + 1)$ is isomorphic with the l_1 space, and the same is true for $C(\omega^\omega + 1)$, but these spaces are not isomorphic, settled the problem if there exist non-isomorphic Banach spaces with isomorphic dual spaces. It was [3] proved (under CH) that if K is Kunen space (constructed under CH compact scattered space) than K^n is hereditarily separable for each $n \in \omega$, but $C(K)$ has no uncountable biorthogonal system. It is possible that spaces of form $C(K)$ where K is compact and scattered can give more examples or counterexamples for open problems.

My current work focuses on studying known results concerning this theme and advanced topics in forcing, combinatorial set theory and Banach space theory. I am a Ph.D. student at Faculty of Mathematics, Informatics and Mechanics of Warsaw University. My advisor is Prof. Piotr Koszmider (Mathematical Institute of Polish Academy of Science and Lodz Technical University).

1. S. Mazurkiewicz, S. Sierpinski, *Contribution a la topologie des ensembles denombrables*, Fun. Math., 1 (1920), 17–27.

2. A. Mostowski, A. Tarski, *Boolesche Ringe mit geordneter Basis*, Fund. Math., 94, 83–92.
3. S. Negrepontis, *Banach spaces and Topology*, in K. Kunen and J. Vaughan, ed., Handbook of Set-Theoretic Topology, North-Holland, (1984), pp. 1045-1142.
4. A. Pełczyński, Z. Semadeni; *Spaces of continuous functions (III) (spaces $C(\Omega)$ for Ω without perfect sets)*, St. Math., 18, 1959.
5. R. Pol, *A function space $C(X)$ which is weakly Lindelöf but not weakly compactly generated.* Studia Math. 64 (1979), no. 3, 279–285.
6. J. Roitman, *Superatomic Boolean algebras*, Handbook of Boolean algebras, Elsevier, 1989.
7. W. Rudin, *Real and Complex Analysis*, McGraw-Hill Companies, 1986.

Miguel Angel Mota
Kurt Gödel Research Center, Vienna, Austria
`motagaytan [at] gmail.com`

Part of the progress in the study of forcing axioms includes the search for restricted forms of these axioms imposing limitations on the size of the real numbers. For example, it was proved by Justin Moore that BPFA implies $2^{\aleph_0} = \aleph_2$. Currently there are several proofs known of this implication (and, more generally, of the weaker fact that PFA implies $2^{\aleph_0} = \aleph_2$), but all of them involve applying the relevant forcing axiom to a partial order collapsing ω_2. Therefore, it becomes natural to ask whether or not the forcing axiom for the class of all proper cardinal–preserving posets, or even the forcing axiom for the class of all proper posets of size $\aleph_1$ (which we will call PFA(ω_1)), implies $2^{\aleph_0} = \aleph_2$.

In a recent work with David Asperó I proved that PFA(ω_1) does not impose any bound on the size of the continuum. The corresponding proof is quite technical and uses some new ideas regarding forcing iteration. Actually we are planning to apply these new tools for proving similar results in the context of small fragments of the P-ideal dichotomy or the Open Coloring Axiom.

Norman Lewis Perlmutter
City University of New York, USA
norman314 [at] gmail.com

I am a fourth year Ph.D. student at the CUNY Graduate Center. My advisor is Joel Hamkins. My main research interests are generalizations of the Kunen inconsistency and generalizations of Solovay's theorem on stationary sets, particularly as they relate to the class of hereditarily ordinal-definable sets, HOD. Some of my work in the first of these areas is joint with Hamkins and Kirmayer.

The Kunen inconsistency states that there is no nontrivial elementary embedding $j : V \to V$. In other words, a Reinhardt cardinal is inconsistent with ZFC. The result is best interpreted by allowing j to be a proper class in the sense of von Neumann- Gödel-Bernays set theory, rather than just a definable class, as otherwise it admits a trivial proof. We are working on generalizing the Kunen inconsistency to preclude nontrivial elementary embeddings between a wide variety of models of ZFC, for instance between V and its forcing extensions, between models one of which is eventually stationary correct in the other, from V to HOD and to gHOD, and from any definable inner model to V. The nonexistence of some of these embeddings was known to Woodin, while others are new results.

In Woodin's proof that there is no nontrivial elementary embedding from V to HOD, he shows that if there is such an embedding, then for a particular cardinal, κ, every V-stationary set $S \subseteq \kappa$ with $S \in HOD$ can be partitioned *in HOD* into κ many disjoint stationary sets. Thinking about this proof lead me to investigate the extent to which this situation occurs without assuming the existence of any sort of elementary embedding. The general pattern of my results has been that if κ is not too large of a large cardinal in HOD, then every V-stationary set in HOD can be partitioned in HOD into many V-stationary sets. The partition can be shown to exist even in some cases where the axiom of choice fails in V, whereas Solovay's theorem requires the axiom of choice. These partitions can then be used to prove theorems constraining elementary embeddings between models of set theory.

Peter Krautzberger
University of Michigan in Ann Arbor, USA
pkrautzb [at] umich.edu

My research interests focus on ultrafilters on semigroups $(S, \cdot)$, most prominently countable ones such as $(\omega, +)$, $(\omega, \cdot)$ and $([\omega]^{<\omega}, \cup)$. The field of algebra in the Stone-Čech compactification studies the extension of semigroup operations to βS; such extensions yield a semigroup operation on βS with idempotent elements and a minimal ideal both of which give rise to algebraic Ramsey-type theorems on the underlying semigroups as well as applications in topological dynamics. (Un)fortunately, the field developed almost without set theoretic "problems", i.e., almost all results are theorems of ZFC.

The classical ultrafilter notions (such as P-points and Q-points) on the other hand are extremely neutral with respect to the algebraic structure, e.g., no sum of two ultrafilters on ω can be a P- or Q-point. The set theoretic techniques related to these classical notions often turn out to be inadequate to attack questions about the ultrafilters relevant to the algebraic structure.

In my thesis my research focus lay on algebraic problems that had only been solved consistently, trying to indentify how set theoretic methods come into the picture. This led to results on union ultrafilters, on the existence of various types of idempotent ultrafilters in different models and on ultrafilters on $(\omega, +)$ with a maximal group $\cong \mathbb{Z}$ (and stronger properties).

During my current DFG fellowship I am working on the reverse situation, studying set theoretic means to attack open "algebraic" questions such as whether every continuous homomorphism $\beta\omega \to \omega^*$

is constant[1] or the existence of an infinite increasing chain of idempotent ultrafilters.[2] Approaching these problems with set theoretic machinery is difficult because the manipulation of idempotent ultrafilters, say, via forcing, has not really been studied before. I'm also interested in applications of idempotent ultrafilters in other set theoretic constructions, e.g., the Mildenberger-Shelah model for $NCF \not\Rightarrow FD$ or Blass's model for $Con(\mathfrak{u} < \mathfrak{g})$.

Finally, thanks to François Dorais I have recently developed an interest in the reverse mathematics of Neil Hindman's Finite Sums Theorem, a crucial tool in this field, as well as other open questions regarding its proofs.

Philipp Lücke
University of Münster, Germany
philipp.luecke [at] uni-muenster.de

My research focuses on the use of set-theoretic methods in the study of infinite groups. Examples of these methods are fine structure theory, forcing and (generalized) descriptive set theory.

Automorphism towers. Given a group G with trivial centre, the group $\mathsf{Aut}(G)$ of automorphisms of G also has trivial centre and, by identifying each element $g \in G$ with the corresponding inner automorphism ι_g defined by $\iota_g(h) = ghg^{-1}$ for all $h \in G$, we may assume that $\mathsf{Aut}(G)$ contains G as a subgroup. We iterate this process to construct the automorphism tower $\langle G_\alpha \mid \alpha \in \mathsf{On}\rangle$ of a centreless group G by setting $G_0 = G$, $G_{\alpha+1} = \mathsf{Aut}(G_\alpha)$ (containing G_α as a subgroup) and $G_\lambda = \bigcup_{\alpha<\lambda} G_\alpha$ for every limit ordinal λ. Simon Thomas showed that for each infinite centreless group G of cardinality κ there exists an ordinal $\alpha < (2^\kappa)^+$ such $G_\alpha = G_\beta$ for all $\beta \geq \alpha$. We call the least such α the *height of the automorphism tower of*

[1]In the 1980s Dona Strauss proved such maps have a finite image.

[2]Here, chain means chain in the partial order of idempotent elements defined as $p \leq q$ iff $p \cdot q = q \cdot p = p$.

G_0 and define τ_κ to be the least upper bound for the heights of automorphism towers of centreless groups of cardinality κ. Thomas' result shows that $\tau_\kappa < (2^\kappa)^+$ holds for every infinite cardinal κ. The following problem is still open: *Find a model M of ZFC and an infinite cardinal $\kappa \in M$ such that it is possible to compute the exact value of τ_κ in M.* I search for better upper bounds for τ_κ using the fine structure theory of $L(\mathcal{P}(\kappa))$ and admissible set theory.

Although the definition of automorphism towers is purely algebraic, it also has a set-theoretic essence, because results of Joel Hamkins and Simon Thomas show that there can be groups whose automorphism tower depends on the model of set theory in which it is computed. I am interested in groups whose automorphism tower can be made arbitrarily tall by forcing with partial orders having certain properties. The question whether such groups exist or can be forced to exist for a given class of partial orders is connected to an answer of the above question.

Descriptive set theory at uncountable cardinals. Given an uncountable regular cardinal κ with $\kappa = \kappa^{<\kappa}$, I study definable subsets of the *Generalized Baire Space* ${}^\kappa\kappa$ and their regularity properties. In particular, I am interested in absoluteness statements for $<\kappa$-closed forcings and definable well-orderings of ${}^\kappa\kappa$.

Automorphisms of ultraproducts of finite symmetric groups. Given a non-principal ultrafilter $\mathcal{U}$ over ω, consider the ultraproduct $S_\mathcal{U} = \prod_\mathcal{U} Sym(n)$ of all finite symmetric groups. If (CH) holds, then the automorphism group of $S_\mathcal{U}$ has cardinality $2^{\aleph_1}$. On the other hand, it is consistent that there is a non-principal ultrafilter $\mathcal{U}$ over ω such that every automorphism of $S_\mathcal{U}$ is inner. It is not known whether there always is a non-principal ultrafilter $\mathcal{U}$ such that $S_\mathcal{U}$ has non-inner automorphisms.

Philipp Schlicht
Rheinische Friedrich-Wilhelms-Universität Bonn, Germany
schlicht [at] math.uni-bonn.de

My research is centered around descriptive set theory and equivalence relations on Polish spaces. One of the main motivations in this area is the calculation of the complexity of definable equivalence relations and the connection with classification problems.

I am especially interested in applications of inner model theory to descriptive set theory. This was the topic of my dissertation, in which I characterized the inner models with representatives in all equivalence classes of thin equivalence relations in a given projective pointclass. I have also worked on the descriptive set theory of the space κ^κ and equivalence relations on κ^κ, where κ is an infinite cardinal with $\kappa^{<\kappa} = \kappa$. With Katie Thompson, we are analyzing the class of trees of a fixed regular size up to strict order preserving maps. More recently, I found a computable version of the Lopez-Escobar theorem, which connects classes of countable structures with the logic $\mathcal{L}_{\omega_1\omega}$. With Frank Stephan, we have computed the possible ranks of linear orders which are isomorphic to a structure recognized by a finite state automaton with running time a limit ordinal α.

Piotr Zakrzewski
University of Warsaw, Poland
piotrzak [at] mimuw.edu.pl

Research interests: combinatorial and descriptive set theory and its connections with other fields of mathematics, specifically measure theory and topology.

Some earlier research: the interplay between algebraic, combinatorial and topological properties of a given action of a group G of measurable transformations of a measurable space $(X, \mathcal{A})$ and the existence and properties of G-invariant measures defined on $\mathcal{A}$ In particular: the way in which invariance of a measure influences the size and the structure of the underlying σ-algebra, and the existence and properties of nonmeasurable sets.

Some current research: special subsets of the reals, σ-ideals on Polish spaces. In particular: a recent joint work with Roman Pol presents an alternative (based on a classical descriptive set theory) proof of the Sabok-Zapletal dichotomy stating that if X is a Polish space and I is a σ-ideal on X generated by closed sets and such that the forcing P_I does not add Cohen reals, then for every Borel set $B \subseteq X$ not in I and every Borel function f from B into a Polish space with all fibers in I there is a G_δ-set $G \subseteq B$ not in I such that $f|G$ is 1-1.

A list of my publications and preprints can be found here: `http://www.mimuw.edu.pl/~piotrzak/publications.html`.

Radek Honzik
Charles University, Czech Republic
`radek.honzik [at] ff.cuni.cz`

Right now I am interested in elementary embeddings (of the hypermeasurable type) and the ways they can be "made nicer" for the given context by means of forcing. For example, given $j : V \to M$ such that $(\kappa^{++})^M = \kappa^{++}$, and M is closed under κ-sequences in V (such an embedding can be obtained from the assumption $o(\kappa) = \kappa^{++}$), one can attempt to gauge the difference between $H(\kappa^{++})^M$ and $H(\kappa^{++})$ by looking at properties of a forcing notion $P \in M$ (typically $P \subset H(\kappa^{++})^M$) in the full universe V (take for example P to be the Cohen forcing at κ^{++} and look at the distributivity of this P in V). This generalizes the question which often occurs in the context of a product forcing: given $P \times Q$, how does P behave in V^Q? Since in $j : V \to M$, V is not a generic extension of M, the behaviour of $P \in M$ in V tends to be more complicated.

One can sometimes show that with a preparatory forcing, a class of certain forcing notions defined in M can be "forced" to behave properly in the full universe. A paradigmatic application of this technique is that $o(\kappa) = \kappa^{++}$ suffices to obtain (to take a specific example) a generic extension where κ is still measurable and $2^\alpha = \alpha^{++}$ for every regular cardinal $\alpha \leq \kappa$ (joint with Sy Friedman).

I am also interested in the ways one can attempt to generalize the concept of properness to larger cardinals (typically inaccessibles).

Raphaël Carroy
Université de Lausanne and Paris 7, France
`Raphael.Carroy [at] unil.ch`

I am interested in descriptive set theory, and more specifically the study of measurable functions. There is two ways to define a hierarchy of functions. On the first hand, the Baire hierarchy of functions is defined inductively from the continuous functions using the taking of pointwise limit of a sequence of functions. On the other hand, the Borel hierarchy of functions is based on the complexity of inverse images of open sets in the Borel hierarchy of sets. Lebesgue has showed an exact correspondence between these two hierarchies.

A space is *polish* whenever it is separable and completely metrizable. In other words, when it admits both a metric such that every cauchy sequence converges and a countable dense subset. Given A and B two polish spaces, a function $f : A \to B$ is *of Baire class one* if f is the pointwise limit of a sequence of continuous functions, or equivalently if the inverse image of an open, or Σ^0_1 subset of B is a countable union of closed sets, or Σ^0_2, in A.

For a specific subset of the first Baire class there is a result binding complexity in terms of inverse images and partitions in continuous functions. Indeed the Jayne-Rogers theorem states that for A and B polish and $f : A \to B$ a function, the inverse image of a Σ^0_2 is Σ^0_2 if and only if there is a countable partition $(A_i)_{i\in\mathbb{N}}$ of A in closed sets such that for all $i \in \mathbb{N}$ the restriction of f to A_i is continuous.

Following this result Andretta proved that for A and B totally disconnected polish spaces, Baire class one functions can be represented as strategies in infinite games. Duparc found a game characterisation for the first Baire class of functions, and Semmes extended those results to the second Baire class.

Is it possible to extend those results to Borel functions of finite rank? Or to any Borel function? Is it possible to refine even more this hierarchy of functions, by looking for example at the Wadge degree of inverse images of open sets? Or by defining a notion of reduction for functions? This is some of the questions I am working on.

1. Kechris, A.: *Classical Descriptive Set Theory.* Springer (1995).
2. Semmes, B.: *A Game for the Borel Functions*, PhD thesis, Universiteit van Amsterdam (2009).

Saeed Ghasemi
York University, Toronto, Canada
saeed.ghas [at] gmail.com

I am a second year PhD student at York University, working on Set Theory under the supervision of Ilijas Farah.

My interest in logic is in Set Theory and Model Theory. During my Masters I was working on extensions of infinitary logics by adding quantifiers based on large cardinals, namely the first weakly compact cardinal. Recently I have been working on some applications of Set Theory to the Theory of C*-algebras. It has been observed that some C*-algebras have really nice properties and behave nicely under some Set Theoretic assumptions. The main theorem that I am interested in is to generalize the following theorem by Ilijas Farah to a larger collection of C*-algebras: “OCA implies that all the automorphisms of the Calkin Algebra are inner.”

I found this Set Theory workshop at Königswinter quite interesting and a good opportunity to learn more about different topics in

Set Theory as well as be around a great community of set theorists. I hope I would get the chance to be there.

Samuel Coskey
Fields Institute, Toronto, Canada
scoskey [at] nylogic.org

My research is in Borel equivalence relations, an area of descriptive set theory. A great reference for this topic is Su Gao's monograph *Invariant Descriptive Set Theory*. The subject begins with the observation that often a classification problem can be identified with an equivalence relation on a standard Borel space (*i.e.*, a Polish space equipped just with its σ-algebra of Borel sets). For instance, each group with domain $\mathbb{N}$ is determined by its group operation, a subset of $\mathbb{N}^3$. Hence, the space of countable groups may be identified with a subset $X_{\mathcal{G}} \subset \mathcal{P}(\mathbb{N}^3)$. Studying the classification problem for countable groups now amounts to studying the isomorphism equivalence relation $\cong_{\mathcal{G}}$ on $X_{\mathcal{G}}$. More generally, we can consider arbitrary equivalence relations on standard Borel spaces.

The central notion is the following comparison of the complexity of equivalence relations, which was introduced by Friedman and Stanley in 1989. If E and F are equivalence relations on standard Borel spaces X and Y, then we say E is *Borel reducible* to F (written $E \leq_B F$) if there is a Borel function $f\colon X \to Y$ satisfying

$$x \; E \; x' \iff f(x) \; F \; f(x') \; .$$

When $E \leq_B F$, then the F-classes Y/F can be used as complete invariants for the classification problem for elements of X up to E. In this sense, $E \leq_B F$ signifies that the classification problem for elements of X up to E is *no harder than* the classification problem for elements of Y up to F.

For my dissertation I studied a classical problem: the classification of torsion-free abelian groups of finite rank. Recently, Hjorth and Thomas showed that $\cong_n <_B \cong_{n+1}$, where $\cong_n$ denotes the isomorphism relation on the collection of torsion-free abelian groups of rank n. Perhaps surprisingly, for $n \geq 2$ this result uses nontrivial techniques from the superrigidity theory for ergodic actions of lattices. In my thesis I considered the *quasi-isomorphism* relations $\sim_n$, and expanding on Thomas's techniques, showed for instance that $\cong_n$ and $\sim_n$ are *Borel incomparable* for $n \geq 3$.

More recently, I have become interested in many other topics in the field. For instance, I briefly studied the classification problem for various families of countable models of Peano arithmetic. I have also recently studied a family of combinatorial properties of countable Borel equivalence relations which have a close connection with the so-called *unions problem.* (This asks whether the increasing union of hyperfinite equivalence relations is again hyperfinite.) Finally, I am interested in broad generalizations of the subject—for instance, what happens if we allow reduction functions computable by an infinite time Turing machine?

Sean Cox
University of Münster, Germany
sean.cox [at] uni-muenster.de

Recently I've focused on strong forcing axioms like PFA and MM, particularly their effect on generic embeddings of V with critical point ω_2. My interest in this subject was motivated by two condensation-like consequences of forcing axioms. Here is a very imprecise formulation of these results (if $M \prec H_\theta$ then $\sigma_M : H_M \to M$ denotes the inverse of the Mostowski collapse of M):

(1) (Foreman [3]) Assume MM. Then there are stationarily many $M \in [H_\theta]^{\omega_1}$ for which H_M is correct about a large portion of $NS \restriction \mathrm{cof}(\omega)$.

(2) (Viale-Weiss [5]) Assume PFA. Then there are stationarily many $M \in [H_\theta]^{\omega_1}$ such that, if $F : [H_\lambda]^{\omega_1} \to V$ is a *slender function* and $F \in M$, then $\sigma_M^{-1}[F(M \cap H_\lambda)]$ is an element of H_M.

In [1] I strengthened Foreman's result and introduced the *Diagonal Reflection Principle (DRP)*, which is a highly simultaneous form of stationary set reflection. Similar results were independently obtained by Viale [4]. DRP has several convenient characterizations; one characterization states that the forcing with positive sets for $NS \restriction \wp_{\omega_2}(\theta)$ has a property resembling but weaker than properness (namely, that stationary subsets of $[\theta]^\omega$ remain stationary in $ult(V, G)$ though not necessarily in $V[G]$). This prompted the natural question of whether PFA can co-exist with ideals on ω_2 whose associated posets are proper; in [2] I showed that this is possible, starting from a superhuge cardinal.

There are natural strengthenings of PFA (which hold in the model from [2]) which imply there are generic embeddings of V where a large portion of the embedding is an element of V. This uses DRP and ideas from [3]. I am currently exploring this further with Matteo Viale and Christoph Weiss, and am also looking into topological applications of DRP.

1. Cox, Sean. The Diagonal Reflection Principle. *submitted.* Preprint available at `http://wwwmath.uni-muenster.de/logik/Personen/Cox/Research`
2. Cox, Sean. PFA and ideals on ω_2 whose associated forcings are proper. *submitted.* Preprint available at `http://wwwmath.uni-muenster.de/logik/Personen/Cox/Research`
3. Foreman, Matthew. Smoke and mirrors: combinatorial properties of small cardinals equiconsistent with huge cardinals. *Adv. Math.*, 222(2):565–595, 2009. ISSN 0001–8708.
4. Viale, Matteo. On the notion of guessing model (preprint). 2010.
5. Viale, Matteo and Weiß, Christoph. The combinatorial essense of supercompactness (preprint). 2010.

Sławomir Solecki
University of Illinois at Urbana-Champaign, USA
`ssolecki [at] math.uiuc.edu`

My mathematical work is frequently motivated by Descriptive Set Theory. Descriptive Set Theory is a branch of mathematical logic studying definable subsets and definable quotients of Polish spaces. Such subsets and quotients occur in a number of areas of mathematics, which makes it possible to apply descriptive set theoretic methods to problems in these areas. Some of my work consists of such applications.

Sławomir Szczepaniak
Polish Academy of Sciences, Poland
`szczepaniak [at] impan.pan.wroc.pl`

My research concentrates on interactions between set theory (or more generally foundations of mathematics) and analysis; I am fond of applications of set theoretical and model theoretical tools to analysis and the theory of Banach spaces especially.

The main object of my investigations is the classical quotient Banach space ℓ_∞/c_0 isomorphically isometric to the space of continuous functions on $\omega^\star$, a Stone space of Boolean algebra $\mathcal{P}(\omega)/\text{fin}$. I am mostly interested in constructing special operators on ℓ_∞/c_0 under PFA or its weaker versions.

Most recently, I have been looking at the possibilities of using ultrafilters and some elements of the Ramsey space theory in order to construct special Banach spaces [1][7]. My current efforts in this topic regard ℓ_p-like Banach spaces possessing unconditional bases with some additional features [5]. Nice reference for some problems connecting Banach spaces and set theory is [2].

1. S.A. Argyros and S. Todorcevic, *Ramsey Methods in Analysis*, Birkhäuser Verlag, Basel (2005).
2. P. Dodos, J. Lopes-Abad and S. Todorcevic, *Banach spaces and Ramsey Theory: some open problems*, Rev. R. Acad. Cien. Serie A Matemáticas 104 (2010), 435-450.
3. R.Frankiewicz, C. Ryll-Nardzewski, S. Szczepaniak, *Some notes on embeddings into the Banach space* ℓ_∞/c_0, preprint
4. R.Frankiewicz, S. Szczepaniak, *Large sets in the sense of the Ellentuck topology does not admit the Kuratowski's partition*, submited
5. P. Hajek, V. Montesinos, J. Vanderwerff, V. Zizler, *Biorthogonal Systems in Banach Spaces*, Springer Verlag, New York, (2007).
6. S. Shelah, *Proper and Improper Forcing*, Springer, (1998)
7. S. Todorcevic, *Introduction to Ramsey spaces*, Ann.Math.St (174), Princeton University Press (2010).

Stefan Geschke
Rheinische Friedrich-Wilhelms-Universität Bonn, Germany
`stefan.geschke [at] hcm.uni-bonn.de`

My research interests are centered around applications of set theory, logic and infinite combinatorics to problems in geometry, Boolean algebras, C^*-algebras, Banach spaces, dynamical systems and measure theory. Another field of interest, related to some of the areas mentioned before, is finite and infinite Ramsey theory.

Currently I am working on problems related to definable graphs and hypergraphs on Polish spaces. Interesting open questions are whether there is a universal clopen graph on the Baire space ω^ω and whether there is a universal clopen graph on a the Cantor space 2^ω.

Definable graphs can be analyzed in terms of natural cardinal invariants such as clique number, chromatic number, Borel chromatic number, and cochromatic number. While there are several results characterizing graphs and hypergraphs for which some of these cardinal invariants are uncountable, most notably the $\mathcal{G}_0$-dichotomy of Kechris, Solecki and Todorcevic about the Borel chromatic number of analytic graphs, apart from some consistency results little is known about how these cardinal invariants relate to the familiar cardinal characteristics of the continuum.

Particular instances of a definable hypergraphs come up in convex geometry, namely the so-called defectedness hypergraphs of closed subsets of $\mathbb{R}^n$. The convex structure of closed subsets of the euclidean plane is fairly well understood by now, but there is no general picture in higher dimensions yet.

I am also thinking about automorphisms of the Boolean algebra $\mathcal{P}(\omega)/\mathtt{fin}$ and of the Calkin algebra $\mathcal{C}$, the quotient of the algebra of bounded operators on a separable Hilbert space modulo the compact operators. Both $\mathcal{P}(\omega)/\mathtt{fin}$ and the Calkin algebra have a natural interesting automorphism, the shift, which we denote by s is both cases. It is known that for example under PFA $(\mathcal{P}(\omega)/\mathtt{fin}, s)$ is not isomorphic to $(\mathcal{P}(\omega)/\mathtt{fin}, s^{-1})$ and $(\mathcal{C}, s)$ is not isomorphic to $(\mathcal{C}, s^{-1})$. It is wide open, however, whether it is consistent that the respective structures are isomorphic. If isomorphism is consistent, then it is natural to conjecture that isomorphism actually follows from CH.

Stuart King
University of Bristol, UK
Stuart.King [at] bristol.ac.uk

I'm currently working on problems in inner model theory. I'm particularly looking at the core model construction in models of ZFC which contain various large cardinals. While I haven't done any work in the area, I'm also interested in class sized forcings.

My general interests include various other areas of set theory, logic and computer science including unprovability theory and reverse mathematics.

Thilo Weinert
Rheinische Friedrich-Wilhelms-Universität Bonn, Germany
weinert [at] math.uni-bonn.de

Currently I am a PhD student of Stefan Geschke in Bonn. I work on a project concerned with cardinal charactaristics that are derived from continuous Ramsey theory. A theorem of Andreas Blass states that if one colours the n-tupels of elements of the Cantor space continuously with m colours, m and n both being finite, then there exists a perfect weakly homogeneous set. "Weakly homogeneous" here means that the colour of an n-tupel only depends on the order of the levels where the branches separate. As an example a tripel can be such that the two leftmost branches separate before the two rightmost do or vice versa. In general an n-tupel has one of $(n-1)!$ possible splitting types.

Now these weakly homogeneous sets generate a σ-ideal and one can ask for its covering number, i.e. the minimal size of a family of weakly homogeneous sets covering the whole space. This is a cardinal characteristic and indeed one that tends to be large. There are two respects in which it is large, its cardinal sucessor has size at least continuum and it is always at least as large as the cofinality of the null ideal and hence at least as large as any cardinal characteristic from Cichoń's diagram. Both facts were found by Stefan Geschke.

Currently it is known that the characteristic for pairs is small, i.e. $\aleph_1$ in the Sacks model and it is known how to separate them from one another.Much more is unknown however.

It is unknown although conjectured that generally the characteristic for n-tupels is small in the Sacks model.
It is unknown how the characteristics relate to characteristics in van Douwen's diagram which do not lie below $\mathfrak{d}$, that is $\mathfrak{a}, \mathfrak{i}, \mathfrak{r}$ and $\mathfrak{u}$.
It is unkown whether such a characteristic can be smaller than the continuum when the latter is say $\aleph_3$.

In the last weeks I also thought a lot about partition relations between countable ordinals, a topic which I consider to be very interesting, this endeavour started some time ago with papers of Erdős, Rado and Specker. Moreover I am quite generally fascinated by countable ordinals.

1. A. Blass, *A partition theorem for perfect sets*, Proc. Am. Math. Soc. **82**(1981), no. 2, 271–277
2. S. Frick, *Continuous Ramsey theory in higher dimensions*, Ph.D. thesis, Freie Universität Berlin (2008)
3. S. Geschke, M. Kojman, W. Kubiś, R. Schipperus, *Convex decompositions in the plane and continuous pair colourings of the irrationals*, Israel Journal of Mathematics, **131**(2002) 285–317
4. S. Geschke, M. Goldstern, M. Kojman, *Continuous Ramsey theory on Polish spaces and covering the plane by functions*, J. Math. Log. **4**(2004), no. 2, 109–145
5. J. Moore, *Open colorings, the continuum and the second uncountable cardinal*, Proc. Am. Math. Soc. **130**(2002), no. 9, 2753–2759
6. Pál Erdős and Richard Rado, *A partition calculus in set theory,* ***Bulletin of the American Mathematical Society*** 62 (1956), 427–489.
7. Ernst Specker, *Teilmengen von Mengen mit Relationen,* ***Commentationes Mathematicae Helvetia,*** 31:302–314, 1957.

Thomas Johnstone
City University of New York, USA
`TJohnstone [at] citytech.cuny.edu`

My research in set theory centers on large cardinals, indestructibility results, forcing axioms, and fragments of ZFC. A recent article, joint with Joel Hamkins, summarizes what we know about indestructibility for a certain kind of large cardinals, called the *strongly unfoldable* cardinals. Another area that I am interested in, are instances of forcing axioms much weaker than PFA, such as its bounded version BPFA, as well as the *Resurrection Axioms*, a new class of forcing axioms that form the center of an forth-coming article of Hamkins and mine. Currently, I am working with Victoria Gitman on obtaining indestructibility results for *Ramsey-like* cardinals; the hope is to also obtain results applicable to Ramsey cardinals. Another project aims to investigate what happens when the Power Set Axiom is removed from the usual ZFC axioms.

Tristan Bice
Kobe University
tristan.bice [at] gmail.com

Much set theory has been done on subsets of ω. Indeed, the whole field of set theory was driven in the early days by the continuum hypothesis, a statement about the cardinality of all such subsets. My research treats l^2 and its closed subspaces as 'quantum' analogs of ω and its subsets, dealing with the whole range of natural analogous questions that arise. In particular, my research deals with projections onto such subspaces in the Calkin algebra of l^2 with their canonical order or, equivalently, with the modulo compact preorder

$$P \leq^* Q \Leftrightarrow P - PQ \text{ is compact},$$

by analogy with subsets of ω preordered by inclusion modulo finite subsets. For starters, it is possible to define many cardinal invariants using projections with this preorder, by analogy with the way classical cardinal invariants can be defined using subsets with the modulo finite preorder. Some ZFC inequalities and consistency results separating these cardinal invariants have been proved, both by myself and others, although much work remains to be done in this area. In a slightly different but related direction, it is possible to define natural forcing notions with projections analogous to those defined with subsets, like Mathias forcing for example. I have defined and proved some basic properties about such forcings, although the structure of such forcing extensions remains, for the most part, unexplored. Finally, in my more recent work, I have been looking at quantum filters of projections and their relation to ultrafilters on ω, in particular under the canonical embedding. As mentioned in [2], maximal quantum filters correspond to pure states on the Calkin algebra, which are important because they give rise to irreducible representations of the Calkin algebra.

1. Andreas Blass, *Combinatorial Cardinal Characteristics of the Continuum*, Handbook of Set Theory (preprint), 24 November (2003).
2. Ilijas Farah and Eric Wofsey, *Set Theory and Operator Algebras*, http://www.math.yorku.ca/ ifarah/Ftp/appalachiannotes-june.pdf, (2009).
3. Eric Wofsey, *$P(\omega)/\mathrm{Fin}$ and Projections in the Calkin Algebra*, arXiv:math.Lo/0702309v1, 12 February (2007).
4. Beatriz Zamora-Aviles, *The Structure of Order Ideals and Gaps in the Calkin Algebra*, PhD Thesis, York University, October (2009).

Vassilis Gregoriades
TU Darmstadt, Germany
`gregoriades [at] mathematik.tu-darmstadt.de`

My research area is descriptive set theory and I am particularly focused on two directions. The first one is effective descriptive set theory (for short effective theory) which combines methods from logic and descriptive set theory and the other direction is applications of descriptive set theory/effective theory in analysis.

The spaces on which effective theory was originally developed are of the form $\omega^k \times \mathcal{N}^m$, where $\mathcal{N} = \omega^\omega$ is the space of all infinite sequences of natural numbers (Baire space). Later on effective theory was extended to the space of the real numbers $\mathbb{R}$ and finally to *recursively presented* Polish spaces with no isolated points, c.f. [4]. My current research on the subject is originated by a construction in my Ph.D. Thesis which assigns to every tree on ω a Polish space $\mathcal{X}^T$. The effective structure of $\mathcal{X}^T$ is depended on the combinatorial properties of $\mathcal{X}^T$. For particular choices of T, well known theorems of effective theory on perfect Polish spaces do fail on the space $\mathcal{X}^T$. In particular there is a recursive tree T such that the space $\mathcal{X}^T$ is uncountable but not Δ^1_1-isomorphic with the Baire space, showing thus that the effective analogue of the well-known statement “every uncountable Polish space is Borel-isomorphic with the Baire space” is not true, c.f. [3]. On the other hand every recursively presented

Polish space is Δ^1_1 isomorphic with a space of the form $\mathcal{X}^T$ and therefore one could say that spaces $\mathcal{X}^T$ are the correct effective analogue of the Baire space. The main question is to study the structure of the spaces $\mathcal{X}^T$ and to classify them up to Δ^1_1-isomorphism.

Effective theory has also applications to other areas of mathematics such as real analysis and Banach space theory. I am particularly interested into theorems which imply the existence of Δ^1_1-points in Polish spaces. The points which are in Δ^1_1 are of special importance as they reduce the complexity of given sets from Σ^1_2 to Π^1_1 and in turn one can apply well-known theorems about co-analytic sets. Also Δ^1_1-points provide Borel-measurable choice functions, c.f. [1] and [2], where no other "classic" i.e., non-effective arguments seem to apply.

1. G. Debs, *Effective properties in compact sets of Borel functions*, Mathematica, **34 (1)**, 1987, 64-68.
2. V. Gregoriades, *A dichotomy result for a pointwise summable sequence of operators*, Annals of Pure and Applied Logic, **160 (2)**, 2009, 154-162.
3. V. Gregoriades, *Effective Descriptive Set Theory and applications in Analysis*, **Ph. D. Thesis**, National and Kapodistrian University of Athens, 2009.
4. Y.N. Moschovakis, *Descriptive Set Theory*, **Studies in Logic and the Foundations of Mathematics**, 100, North-Holland Publishing Co., 1980.

Vera Fischer
Kurt Gödel Research Center, Vienna, Austria
`vera.fischer [at] univie.ac.at`

My main interests are in infinitary combinatorics and forcing. I have also interests in definability, as well as applications of set theoretic techniques to analysis and topology.

In the last few years, I have been working on obtaining various consistency results, requiring continuum greater than or equal to $\aleph_3$ (see [5], [2], [6]). In more recent work, I consider the existence of various combinatorial objects on the real line in the presence of a projective wellorder of the reals (see [3], [4]). Also, I have interests in non-linear iterations (see [1]), in particular template forcing (see [8]), combinatorics of uncountable cardinals and in some questions concerning large cardinals and forcing.

1. J. Brendle *Mad families and ultrafilters* Acta Universitatis Carolinae, Mathematicae et Physica **49** (2007), 19-35.
2. J. Brendle, V. Fischer *Mad families, splitting families and large continuum*, Journal of Symbolic Logic, 76, 1, March 2011, pp.198-208.
3. V. Fischer, S. D. Friedman *Cardinal characteristics and projective well-orders* Annals of Pure and Applied Logic 161 (2010), 916-922.
4. V. Fischer, S. D. Friedman, L. Zdomskyy *Projective wellordrs and mad families with large continuum* submitted.
5. V. Fischer, J. Steprāns *The consistency of* $\mathfrak{b} = \kappa < \mathfrak{s} = \kappa^+$ Fundamenta Mathematicae **201** (2008).
6. V. Fischer, J. Steprāns *Further combinatorial properties of Cohen forcing* RIMS Conference proceedings in Combinatorial and Descriptive Set Theory, Kyoto, 2008.
7. V. Fischer, A. Törnquist *A co-analytic maximal set of orthogonal measures* Journal of Symbolic Logic, 75, 4, pp. 1403 - 1414.
8. S. Shelah *Two cardinal invariants of the continuum* ($\mathfrak{d} < \mathfrak{a}$) *and FS linearly ordered iterated forcing* Acta Math. **192** (2004), 187-223.

Víctor Torres Pérez

Kurt Gödel Research Center, Vienna, Austria

victor.torres.perez [at] gmail.com

Recall that the compactness principle for infinitary logics was introduced in the paper by Erdös and Tarski in 1943 [1] motivated of course by Gödel's compactness theorem for first-order logic. We would like to concentrate on a rather weak form of this compactness principle through a combinatorial principle called Rado's Conjecture, RC, first considered by Richard Rado (see, for example [4]). It is a compactness principle for the chromatic number of intersection graphs on families of intervals of linearly ordered sets. It states that if such a graph is not countably chromatic then it contains a subgraph of cardinality $\aleph_1$ which is also not countably chromatic. The consistency of Rado's Conjecture has been established by Todorcevic in 1983 [5] using the consistency of the existence of a supercompact cardinal. In 1991, Todorcevic showed that RC $\longrightarrow (\forall\theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \theta^{\aleph_0} = \theta$ (see [6]). So, in particular RC implies the Singular Cardinals Hypothesis. It is possible to show that combining the arguments of Todorcevic [5] and of Foreman-Magidor-Shelah [3] one obtains that RC is also relatively consistent with the assertion that the ideal NS_{ω_1} of non-stationary subsets of ω_1 is saturated. On the other hand, Feng proved that Rado's Conjecture implies the presaturation of the ideal NS_{ω_1} [2]. Thus, it is natural to examine whether RC supplemented by the saturation of NS_{ω_1} would also give us stronger consequences for cardinal arithmetic. In short, we would like to show that $\mathrm{RC} + \mathrm{sat}(\mathrm{NS}_{\omega_1}) = \aleph_2 \longrightarrow (\forall\theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \theta^{\aleph_1} = \theta$. In fact, we would like show that this assumption will give us a bit stronger result, $\mathrm{RC} + \mathrm{sat}(\mathrm{NS}_{\omega_1}) = \aleph_2 \longrightarrow (\forall\theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \diamondsuit_{[\theta]^{\omega_1}}$.

1. P. Erdös and A. Tarski. On families of mutually exclusive sets. *Ann. of Math. (2)*, 44:315–329, 1943.
2. Qi Feng. Rado's conjecture and presaturation of the nonstationary ideal on ω_1. *J. Symbolic Logic*, 64(1):38–44, 1999.
3. M. Foreman, M. Magidor, and S. Shelah. Martin's maximum, saturated ideals, and nonregular ultrafilters. I. *Ann. of Math. (2)*, 127(1):1–47, 1988.
4. Richard Rado. Theorems on intervals of ordered sets. *Discrete Math.*, 35:199–201, 1981.
5. S. Todorčević. On a conjecture of R. Rado. *J. London Math. Soc. (2)*, 27(1):1–8, 1983.
6. Stevo Todorčević. Conjectures of Rado and Chang and cardinal arithmetic. In *Finite and infinite combinatorics in sets and logic (Banff, AB, 1991)*, volume 411 of *NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci.*, pages 385–398. Kluwer Acad. Publ., Dordrecht, 1993.

Vincenzo Dimonte
Kurt Gödel Research Center, Vienna, Austria
`vincenzo.dimonte [at] gmail.com`

I0, i.e the existence of an elementary embedding j from $L(V_{\lambda+1})$ to itself, for some $\lambda > crt(j)$, has been considered for some time the strongest large cardinal hypothesis. The appeal of I0 consists in the particular features shared with $AD^{L(\mathbb{R})}$, like measurability or a generalization of the Coding Lemma. Woodin in his future latest book introduced new large cardinal hypotheses, stronger than I0, and proved similar results of affinity. The idea is that between $L(V_{\lambda+1})$ and $L(V_{\lambda+2})$ there are a lot of intermediate steps, namely $L(N)$ with $V_{\lambda+1} \subset N \subset V_{\lambda+2}$, with the particular case $L(X, V_{\lambda+1})$ with $X \subset V_{\lambda+1}$. Trying to find an equivalent of $AD_{\mathbb{R}}$, Woodin defined a sequence of $E^0_\alpha(V_{\lambda+1})$, in a similar way of the minimum model of $AD_{\mathbb{R}}$, such that $V_{\lambda+1} \subseteq E^0_\alpha(V_{\lambda+1}) \subseteq V_{\lambda+2}$, $L(E^0_\alpha(V_{\lambda+1})) \cap V_{\lambda+2} = E^0_\alpha(V_{\lambda+1})$, there always exists an elementary embedding from $L(E^0_\alpha(V_{\lambda+1}))$ to itself and when the sequence ends in the "right way" (if that is consistent), then $L(\bigcup E^0_\alpha(V_{\lambda+1}))$ has characteristics similar to a model that satisfies $AD_{\mathbb{R}}$.

My attention is focused on the properties of the elementary embeddings from $L(E^0_\alpha(V_{\lambda+1}))$ to itself, expecially when $L(E^0_\alpha(V_{\lambda+1})) \vDash V = HOD_{V_{\lambda+1}}$. One of the key properties in this setting is *properness*, that is a fragment of the Axiom of Replacement for the elementary embedding. Properness implies iterability and many other similarities wirh $AD^{L(\mathbb{R})}$, but it seems almost trivial, since the most immediate examples of elementary embeddings satisfy it. In my PhD thesis, in collaboration with Woodin, I've proved that in fact if the sequence is long enough (for example when it ends in the right way) then there exists an α such that every elementary embedding from $L(E^0_\alpha(V_{\lambda+1}))$ into itself is not proper. It is also possible to find an $L(E^0_\beta(V_{\lambda+1}))$ that contains both proper and non-proper elementary embeddings, so properness is not a property directly implied by the model.

In the future I want to approach the hypothesis that the $E^0_\alpha(V_{\lambda+1})$-sequence does *not* end in the right way. This would imply that the $E^0_\alpha(V_{\lambda+1})$-sequence is a standard representative for all the $L(X, V_{\lambda+1})$

that contain elementary embedding, and a preliminary study seems to indicate that this will give results on the properness of elementary embeddings from $L(X, V_{\lambda+1})$ to itself, that is still an open problem. Since this is still a virgin territory (even because it's based on unpublished results), the possible outcomings are many.

Wolfgang Wohofsky
TU Wien, Austria
`wolfgang.wohofsky [at] gmx.at`

I am a PhD student mainly interested in (iterated) forcing and its applications to set theory of the reals; in particular, I am studying questions about small subsets of the real line and (variants of) the Borel Conjecture.

The *Borel Conjecture* (BC) is the statement that there are no uncountable strong measure zero sets (a set X is *strong measure zero* if for any sequence of ε_n's, X can be covered by intervals I_n of length ε_n, or, equivalently, if it can be translated away from each meager set). The *dual Borel Conjecture* (dBC) is the analogous statement about *strongly meager* sets (the sets which can be translated away from each measure zero set). Both BC and dBC fail under CH. In 1976, Laver [4] showed that BC is consistent (by a countable support iteration of Laver forcing of length ω_2). Carlson [2] showed that dBC is consistent (by a finite support iteration of Cohen forcing of length ω_2). What about BC + dBC?

Together with my advisor Martin Goldstern, Jakob Kellner and Saharon Shelah, I have been working on the proof of the following theorem (see [3]):

> There is a model of ZFC in which both the Borel Conjecture and the dual Borel Conjecture hold, i.e., Con(BC + dBC).

One of the difficulties in the proof is the fact that one is forced to obtain dBC *without adding Cohen reals* since Cohen reals inevitably destroy BC. This was first done by Bartoszyński and Shelah [1] using

Shelah's non-Cohen oracle-c.c. framework from [5]. For this reason I made myself acquainted with this framework even though it turned out to be not directly applicable to the problem Con(BC + dBC). Nevertheless I still plan to write up a more digestible version of the non-Cohen oracle-c.c. framework.

Some time ago, I also started to investigate another variant of the Borel Conjecture, which I call the *Marczewski Borel Conjecture* (MBC). It is the assertion that there are no uncountable sets in s_0^*, where s_0^* is the collection of those sets which can be translated away from each set in the *Marczewski ideal* s_0 (the Marczewski ideal s_0 is related to Sacks forcing: a set X is in s_0 if each perfect set contains a perfect subset disjoint from X). So MBC is the analogue to BC (dBC) with meager (measure zero) replaced by s_0 in its definition. The question arises whether MBC is consistent (the negation of MBC is consistent).

I do not know, but while exploring the family s_0^* under CH, I obtained the following result. Let's call $\mathcal{I} \subseteq \mathcal{P}(2^\omega)$ a *Sacks dense ideal* if

- $\mathcal{I}$ is a (non-trivial) translation-invariant σ-ideal
- $\mathcal{I}$ is "dense in Sacks forcing": each perfect set P contains a perfect subset $Q \subseteq P$ which belongs to $\mathcal{I}$.

Then the following holds:

> Assume CH. Then s_0^* is contained in every Sacks dense ideal $\mathcal{I}$.

So the question is whether we can (at least consistently) find "many Sacks dense ideals" (under CH). The meager sets as well as the measure zero sets form Sacks dense ideals, whereas the strong measure zero sets do not. Nevertheless the strong measure zero sets can be "approximated from above", meaning that each set in the intersection of all Sacks dense ideals (and hence each set in s_0^*) is strong measure zero (and also perfectly meager).

Website: http://www.wohofsky.eu/math/

1. Tomek Bartoszyński and Saharon Shelah. Dual Borel Conjecture and Cohen reals. *Journal of Symbolic Logic*, 75:1293–1310, 2010.

2. Timothy J. Carlson. Strong measure zero and strongly meager sets. *Proc. Amer. Math. Soc.*, 118(2):577–586, 1993.
3. Martin Goldstern, Jakob Kellner, Saharon Shelah, and Wolfgang Wohofsky. Borel Conjecture and dual Borel Conjecture. *Preprint.*
4. Richard Laver. On the consistency of Borel's conjecture. *Acta Math.*, 137(3-4):151–169, 1976.
5. Saharon Shelah. Non-Cohen Oracle c.c.c. *Journal of Applied Analysis*, 12:1–17, 2006. math.LO/0303294.

Zoltán Vidnyánszky
Eötvös Lóránd University, Budapest, Hungary
`vidnyanszkyz [at] gmail.com`

I am a second year student of MSc in Mathematics at the Eötvös Lóránd University, and I am interested in set theory, real analysis and topology. I wrote my BSc thesis under the supervision of Péter Komjáth about certain problems in descriptive set theory. The main question was that whether there exists an uncountable set intersecting every nice arc in countably many points. It turned out that this question is closely related to SOCA and forcing and was answered in the later published article of Hart and Kunen. In the past two years I did some research in effective descriptive set theory. I passed some courses in the topics of forcing, basic and advanced set theory and PCF theory (tought by Lajos Soukup), tried to understand the article which proves the consistency of SOCA (Uri Abraham, Matatyahu Rubin, Saharon Shelah: On the Consistency of Some Partition Theorems for Continuous Colorings). Now I am especially intrested in V=L and constructibility.

Zu Yao Teoh
University of Manchester, UK
zuyao.teoh [at] postgrad.manchester.ac.uk

My interest is currently in descriptive set theory (effective theory) and inner models of set theory. In my master's thesis, I wrote a dissertation in the topic of determinacy, in which I surveyed the basic and classical results of descriptive set theory up to proofs of projective determinacy. I first surveyed the inconsistency of the axiom of determinacy AD with the axiom of choice AC and the consistency of AD with the countable axiom of choice on the reals $AC_\omega(\mathbb{R})$. Next, I surveyed the existence of sets that do not possess the three regularity properties 1. measurability, 2. perfect set property, and 3. Property of Baire assuming the axiom of choice. Then, I showed that analytic sets possess these properties. Following the usual theme, I showed that assuming AD, every set of reals possesses the three properties mentioned and I briefly showed that the weaker version Projective Determinacy PD proves that every projective set possesses the properties. Finally, I wrote of the proofs of (a) Borel Determinacy (including the determinacy of open sets and "finitely supported sets" [I. Neeman]), (b) Analytic Determinacy assuming a Ramsey cardinal, and (c) Projective Determinacy given the existence of infinitely many Woodin cardinals with a measurable cardinal above them. The exposition of (b) and (c) were written with the assumption of some properties of cardinals. In (b), I have called a cardinal with the property which entails analytic determinacy the "Ramsey property" (I do not know if this is standard) and stated the corollary that the existence of a measurable cardinals implies analytic determinacy as measurable cardinals possess the Ramsey property (not proved). In (c), the exposition hinges upon the proofs of the following:

(1) A κ-homogeneously Suslin set is determined.
(2) Every projective set is homogeneously Suslin assuming the existence of (enough) Woodin cardinals.

There are related topics which I am interested in and I am studying on. For instance, it has bothered me for quite a while as to which ordinal in the submodel or extension of a ground model (of set theory) correspond to in the ground model, particularly in cases where the submodel is simple in some sense, such as an inner model. A well known result states that the uncountable cardinals of a ground model are inaccessibles in L. (Cf. Corollary 18.3 of [1].) This means that the uncountable cardinals between two successive inaccessibles in L occur among the limit ordinals between two successive uncountable cardinals of the ground model. I am interested to find out more about such connections. In addition, problems in effective descriptive set theory are of interest, too; in particular, in conjunction with the relation between ordinals between ground models and submodels, I am looking at the method used in the proof of a connected theorem by Groszek and Slaman [2] to see if the method can be applied to shed light on the question above. The theorem states that every perfect set has a nonconstructible element if a nonconstructible real exists.

1. Jech, Thomas **Set theory.** The third millennium edition, revised and expanded. Springer Monographs in Mathematics. *Springer-Verlag, Berlin, 2003.* xiv+769 pp. ISBN: 3-540-44085-2
2. Groszek, Marcia J.; Slaman, Theodore A. **A basis theorem for perfect sets.** *Bull. Symbolic Logic 4 (1998), no. 2, 204-209*, 03E45

List of participants

Alexander Primavesi (Norwich)
Ali Enayat (Washington)
Allard van Veen (Bonn)
Andrea Medini (Madison)
Andrew Brooke-Taylor (Bristol)
Anne Fernengel (Bonn)
Assaf Rinot (Beer Sheva)
Brent Cody (New York)
Carolin Antos-Kuby (Vienna)
Carsten Szardenings (Münster)
Christoph Weiss (München)
Daisuke Ikegami (Helsinki)
Daniel Alberto Donado Rincon (Bogota)
Dániel Tamás Soukup (Budapest)
David Chodounsky (Prague)
David J. Fernández Bretón (Toronto)
David Schrittesser (Bonn)
Diana Carolina Montoya Amaya (Bogota)
Dilip Raghavan (Toronto)
Dominik Adolf (Münster)
Edoardo Rivello (Torino)
Erin Carmody (New York)
Fabiana Castiblanco (Sao Paulo)
Gido Scharfenberger-Fabian (Greifswald)
Giorgio Audrito (Turin)
Giorgio Laguzzi (Vienna)
Giorgio Venturi (Paris)
Grigor Sargsyan (Los Angeles)
Hiroaki Minami (Vienna)
Hossein Lamei Ramandi (Tehran)
Ioanna Matilde Dimitríou (Bonn)
Jip Veldman (Bonn)
Joel David Hamkins (New York)
Jonathan L. Verner (Prague)

Juris Steprans (Toronto)
Katie Thompson (Vienna)
Kevin Fournier (Lausanne)
Kostas Tsaprounis (Barcelona)
Lauri Tuomi (Paris)
Liuzhen Wu (Vienna)
Luís Pereira (Lisbon)
Luz María García Ávila (Barcelona)
Marcin Sabok (Urbana-Champaign)
Marek Wyszkowski (Kiel)
Marios Koulakis (Münster)
Matteo Viale (Torino)
Mayra Montalvo Ballesteros (Leeds)
Michal Korch (Warsaw)
Miguel Angel Mota (Vienna)
Norman Lewis Perlmutter (New York)
Peter Koepke (Bonn)
Peter Krautzberger (Michigan)
Philipp Lücke (Münster)
Philipp Schlicht (Bonn)
Piotr Zakrzewski (Warsaw)
Radek Honzik (Prague)
Raimund Fiedler (Bonn)
Raphaël Carroy (Lausanne and Paris)
Saeed Ghasemi (Toronto)
Samuel Coskey (Toronto)
Sean Cox (Münster)
Sławomir Solecki (Urbana-Champaign)
Sławomir Szczepaniak (Wroclaw)
Stefan Geschke (Bonn)
Stefan Knauf (Bonn)
Stuart King (Bristol)
Thilo Weinert (Bonn)
Thomas Johnstone (New York)
Tristan Bice (Kobe)
Vassilis Gregoriades (Darmstadt)
Vera Fischer (Vienna)
Víctor Torres Pérez (Vienna)

Vincenzo Dimonte (Vienna)
Wolfgang Wohofsky (Vienna)
Zoltán Vidnyánszky (Budapest)
Zu Yao Teoh (Manchester)

Acknowledgements

This workshop is funded by the Hausdorff Center for Mathematics at the University of Bonn, the European Science Foundation through the INFTY research network, and the Association for Symbolic Logic. We are grateful to these organizations for their support.

Practical information

Group picture. The group picture is going to be taken on Tuesday 22 March just before lunch, outside the building.

Excursion. On Wednesday 23 March, we plan to walk to the top of Petersberg if the weather allows for it. It takes an hour to reach the top. We will leave from the entrance of the conference center at 15:30.

Social dinner. 19:30 Thursday 24 March
Taste of India (North Indian)
Rheingasse 13, Bonn
http://www.tasteofindia.de/nord.html

On Thursday at 18:40, we will meet at the entrance of the conference building, and at 18:59 take U-Bahn 66 from the stop Longenburg, Königswinter close to the conference center to the stop Universität/Markt, Bonn at 19:21. At this time of day, line 66 goes every 10 minutes. Later in the evening, line 66 goes every half hour, with the last trip departing from Universität/Markt at 1:16.

Registration fee. The registration fee includes accommodation and meals. Not included are drinks other than tea, coffee, and water, and the social dinner on Thursday evening.