RESEARCH STATEMENT

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Recall that the compactness principle for infinitary logics was introduced in the paper by Erdös and Tarski in 1943 [1] motivated of course by Gödel's compactness theorem for first-order logic. We would like to concentrate on a rather weak form of this compactness principle through a combinatorial principle called Rado's Conjecture, RC, first considered by Richard Rado (see, for example [4]). It is a compactness principle for the chromatic number of intersection graphs on families of intervals of linearly ordered sets. It states that if such a graph is not countably chromatic then it contains a subgraph of cardinality $\aleph_1$ which is also not countably chromatic. The consistency of Rado's Conjecture has been established by Todorcevic in 1983 [5] using the consistency of the existence of a supercompact cardinal. In 1991, Todorcevic showed that RC $\longrightarrow (\forall\theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \theta^{\aleph_0} = \theta$ (see [6]). So, in particular RC implies the Singular Cardinals Hypothesis. It is possible to show that combining the arguments of Todorcevic [5] and of Foreman-Magidor-Shelah [3] one obtains that RC is also relatively consistent with the assertion that the ideal NS_{ω_1} of non-stationary subsets of ω_1 is saturated. On the other hand, Feng proved that Rado's Conjecture implies the presaturation of the ideal NS_{ω_1} [2]. Thus, it is natural to examine whether RC supplemented by the saturation of NS_{ω_1} would also give us stronger consequences for cardinal arithmetic. In short, we would like to show that $\mathrm{RC} + \mathrm{sat}(\mathrm{NS}_{\omega_1}) = \aleph_2 \longrightarrow (\forall\theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \theta^{\aleph_1} = \theta$. In fact, we would like show that this assumption will give us a bit stronger result, $\mathrm{RC} + \mathrm{sat}(\mathrm{NS}_{\omega_1}) = \aleph_2 \longrightarrow (\forall\theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \diamondsuit_{[\theta]^{\omega_1}}$.

References

[1] P. Erdös and A. Tarski. On families of mutually exclusive sets. *Ann. of Math. (2)*, 44:315–329, 1943.

[2] Qi Feng. Rado's conjecture and presaturation of the nonstationary ideal on ω_1. *J. Symbolic Logic*, 64(1):38–44, 1999.

[3] M. Foreman, M. Magidor, and S. Shelah. Martin's maximum, saturated ideals, and nonregular ultrafilters. I. *Ann. of Math. (2)*, 127(1):1–47, 1988.

[4] Richard Rado. Theorems on intervals of ordered sets. *Discrete Math.*, 35:199–201, 1981.

[5] S. Todorčević. On a conjecture of R. Rado. *J. London Math. Soc. (2)*, 27(1):1–8, 1983.

[6] Stevo Todorčević. Conjectures of Rado and Chang and cardinal arithmetic. In *Finite and infinite combinatorics in sets and logic (Banff, AB, 1991)*, volume 411 of *NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci.*, pages 385–398. Kluwer Acad. Publ., Dordrecht, 1993.