Research Statement for Young Set Theory Workshop
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My research area is descriptive set theory and I am particularly focused on two directions. The first one is effective descriptive set theory (for short effective theory) which combines methods from logic and descriptive set theory and the other direction is applications of descriptive set theory/effective theory in analysis.

The spaces on which effective theory was originally developed are of the form $\omega^k \times \mathcal{N}^m$, where $\mathcal{N} = \omega^\omega$ is the space of all infinite sequences of natural numbers (Baire space). Later on effective theory was extended to the space of the real numbers $\mathbb{R}$ and finally to *recursively presented* Polish spaces with no isolated points, c.f. [4]. My current research on the subject is originated by a construction in my Ph.D. Thesis which assigns to every tree on ω a Polish space $\mathcal{X}^T$. The effective structure of $\mathcal{X}^T$ is depended on the combinatorial properties of $\mathcal{X}^T$. For particular choices of T, well known theorems of effective theory on perfect Polish spaces do fail on the space $\mathcal{X}^T$. In particular there is a recursive tree T such that the space $\mathcal{X}^T$ is uncountable but not Δ^1_1-isomorphic with the Baire space, showing thus that the effective analogue of the well-known statement "every uncountable Polish space is Borel-isomorphic with the Baire space" is not true, c.f. [3]. On the other hand every recursively presented Polish space is Δ^1_1 isomorphic with a space of the form $\mathcal{X}^T$ and therefore one could say that spaces $\mathcal{X}^T$ are the correct effective analogue of the Baire space. The main question is to study the structure of the spaces $\mathcal{X}^T$ and to classify them up to Δ^1_1-isomorphism.

Effective theory has also applications to other areas of mathematics such as real analysis and Banach space theory. I am particularly interested into theorems which imply the existence of Δ^1_1-points in Polish spaces. The points which are in Δ^1_1 are of special importance as they reduce the complexity of given sets from Σ^1_2 to Π^1_1 and in turn one can apply well-known theorems about co-analytic sets. Also Δ^1_1-points provide Borel-measurable choice functions, c.f. [1] and [2], where no other "classic" i.e., non-effective arguments seem to apply.

References

[1] G. Debs, *Effective properties in compact sets of Borel functions*, Mathematica, **34 (1)**, 1987, 64-68.

[2] V. Gregoriades, *A dichotomy result for a pointwise summable sequence of operators*, Annals of Pure and Applied Logic, **160 (2)**, 2009, 154-162.

[3] V. Gregoriades, *Effective Descriptive Set Theory and applications in Analysis*, **Ph. D. Thesis**, National and Kapodistrian University of Athens, 2009.

[4] Y.N. Moschovakis, *Descriptive Set Theory*, **Studies in Logic and the Foundations of Mathematics**, 100, North-Holland Publishing Co., 1980.