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Much set theory has been done on subsets of $\omega$. Indeed, the whole field of set theory was driven in the early days by the continuum hypothesis, a statement about the cardinality of all such subsets. My research treats $l^2$ and its closed subspaces as 'quantum' analogs of $\omega$ and its subsets, dealing with the whole range of natural analogous questions that arise. In particular, my research deals with projections onto such subspaces in the Calkin algebra of $l^2$ with their canonical order or, equivalently, with the modulo compact preorder

$$P \leq^* Q \Leftrightarrow P - PQ \text{ is compact},$$

by analogy with subsets of $\omega$ preordered by inclusion modulo finite subsets. For starters, it is possible to define many cardinal invariants using projections with this preorder, by analogy with the way classical cardinal invariants can be defined using subsets with the modulo finite preorder. Some ZFC inequalities and consistency results separating these cardinal invariants have been proved, both by myself and others, although much work remains to be done in this area. In a slightly different but related direction, it is possible to define natural forcing notions with projections analogous to those defined with subsets, like Mathias forcing for example. I have defined and proved some basic properties about such forcings, although the structure of such forcing extensions remains, for the most part, unexplored. Finally, in my more recent work, I have been looking at quantum filters of projections and their relation to ultrafilters on $\omega$, in particular under the canonical embedding. As mentioned in [2], maximal quantum filters correspond to pure states on the Calkin algebra, which are important because they give rise to irreducible representations of the Calkin algebra.

# References

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[4] Beatriz Zamora-Aviles, *The Structure of Order Ideals and Gaps in the Calkin Algebra*, PhD Thesis, York University, October (2009).