Research statement

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Abstract

On what I am doing.

Currently I am a PhD student of Stefan Geschke in Bonn. I work on a project concerned with cardinal charactaristics that are derived from continuous Ramsey theory. A theorem of Andreas Blass states that if one colours the n-tupels of elements of the Cantor space continuously with m colours, m and n both being finite, then there exists a perfect weakly homogeneous set. "Weakly homogeneous" here means that the colour of an n-tupel only depends on the order of the levels where the branches separate. As an example a tripel can be such that the two leftmost branches separate before the two rightmost do or vice versa. In general an n-tupel has one of $(n-1)!$ possible splitting types.

Now these weakly homogeneous sets generate a σ-ideal and one can ask for its covering number, i.e. the minimal size of a family of weakly homogeneous sets covering the whole space. This is a cardinal characteristic and indeed one that tends to be large. There are two respects in which it is large, its cardinal sucessor has size at least continuum and it is always at least as large as the cofinality of the null ideal and hence at least as large as any cardinal characteristic from Cichoń's diagram. Both facts were found by Stefan Geschke.

Currently it is known that the characteristic for pairs is small, i.e. $\aleph_1$ in the Sacks model and it is known how to separate them from one another.Much more is unknown however.

It is unknown although conjectured that generally the characteristic for n-tupels is small in the Sacks model.
It is unknown how the characteristics relate to characteristics in van Douwen's diagram which do not lie below $\mathfrak{d}$, that is $\mathfrak{a}, \mathfrak{i}, \mathfrak{r}$ and $\mathfrak{u}$.
It is unkown whether such a characteristic can be smaller than the continuum when the latter is say $\aleph_3$.

In the last weeks I also thought a lot about partition relations between countable ordinals, a topic which I consider to be very interesting, this endeavour started some time ago with papers of Erdős, Rado and Specker. Moreover I am quite generally fascinated by countable ordinals.

References

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