STATEMENT OF RESEARCH INTERESTS AND PLANS

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My research interests are centered around applications of set theory, logic and infinite combinatorics to problems in geometry, Boolean algebras, C^*-algebras, Banach spaces, dynamical systems and measure theory. Another field of interest, related to some of the areas mentioned before, is finite and infinite Ramsey theory.

Currently I am working on problems related to definable graphs and hypergraphs on Polish spaces. Interesting open questions are whether there is a universal clopen graph on the Baire space ω^ω and whether there is a universal clopen graph on a the Cantor space 2^ω.

Definable graphs can be analyzed in terms of natural cardinal invariants such as clique number, chromatic number, Borel chromatic number, and cochromatic number. While there are several results characterizing graphs and hypergraphs for which some of these cardinal invariants are uncountable, most notably the $\mathcal{G}_0$-dichotomy of Kechris, Solecki and Todorcevic about the Borel chromatic number of analytic graphs, apart from some consistency results little is known about how these cardinal invariants relate to the familiar cardinal characteristics of the continuum.

Particular instances of a definable hypergraphs come up in convex geometry, namely the so-called defectedness hypergraphs of closed subsets of $\mathbb{R}^n$. The convex structure of closed subsets of the euclidean plane is fairly well understood by now, but there is no general picture in higher dimensions yet.

I am also thinking about automorphisms of the Boolean algebra $\mathcal{P}(\omega)/\mathtt{fin}$ and of the Calkin algebra $\mathcal{C}$, the quotient of the algebra of bounded operators on a separable Hilbert space modulo the compact operators. Both $\mathcal{P}(\omega)/\mathtt{fin}$ and the Calkin algebra have a natural interesting automorphism, the shift, which we denote by s is both cases. It is known that for example under PFA $(\mathcal{P}(\omega)/\mathtt{fin}, s)$ is not isomorphic to $(\mathcal{P}(\omega)/\mathtt{fin}, s^{-1})$ and $(\mathcal{C}, s)$ is not isomorphic to $(\mathcal{C}, s^{-1})$. It is wide open, however, whether it is consistent that the respective structures are isomorphic. If isomorphism is consistent, then it is natural to conjecture that isomorphism actually follows from CH.

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