RESEARCH STATEMENT - PHILIPP LÜCKE

My research focuses on the use of set-theoretic methods in the study of infinite groups. Examples of these methods are fine structure theory, forcing and (generalized) descriptive set theory.

Automorphism towers. Given a group G with trivial centre, the group $\mathsf{Aut}(G)$ of automorphisms of G also has trivial centre and, by identifying each element $g \in G$ with the corresponding inner automorphism ι_g defined by $\iota_g(h) = ghg^{-1}$ for all $h \in G$, we may assume that $\mathsf{Aut}(G)$ contains G as a subgroup. We iterate this process to construct the automorphism tower $\langle G_\alpha \mid \alpha \in \mathsf{On}\rangle$ of a centreless group G by setting $G_0 = G$, $G_{\alpha+1} = \mathsf{Aut}(G_\alpha)$ (containing G_α as a subgroup) and $G_\lambda = \bigcup_{\alpha<\lambda} G_\alpha$ for every limit ordinal λ. Simon Thomas showed that for each infinite centreless group G of cardinality κ there exists an ordinal $\alpha < (2^\kappa)^+$ such $G_\alpha = G_\beta$ for all $\beta \geq \alpha$. We call the least such α the *height of the automorphism tower of* G_0 and define τ_κ to be the least upper bound for the heights of automorphism towers of centreless groups of cardinality κ. Thomas' result shows that $\tau_\kappa < (2^\kappa)^+$ holds for every infinite cardinal κ. The following problem is still open: *Find a model M of ZFC and an infinite cardinal $\kappa \in M$ such that it is possible to compute the exact value of τ_κ in M.* I search for better upper bounds for τ_κ using the fine structure theory of $L(\mathcal{P}(\kappa))$ and admissible set theory.

Although the definition of automorphism towers is purely algebraic, it also has a set-theoretic essence, because results of Joel Hamkins and Simon Thomas show that there can be groups whose automorphism tower depends on the model of set theory in which it is computed. I am interested in groups whose automorphism tower can be made arbitrarily tall by forcing with partial orders having certain properties. The question whether such groups exist or can be forced to exist for a given class of partial orders is connected to an answer of the above question.

Descriptive set theory at uncountable cardinals. Given an uncountable regular cardinal κ with $\kappa = \kappa^{<\kappa}$, I study definable subsets of the *Generalized Baire Space* ${}^\kappa\kappa$ and their regularity properties. In particular, I am interested in absoluteness statements for $<\kappa$-closed forcings and definable well-orderings of ${}^\kappa\kappa$.

Automorphisms of ultraproducts of finite symmetric groups. Given a non-principal ultrafilter $\mathcal{U}$ over ω, consider the ultraproduct $S_\mathcal{U} = \prod_\mathcal{U} Sym(n)$ of all finite symmetric groups. If (CH) holds, then the automorphism group of $S_\mathcal{U}$ has cardinality $2^{\aleph_1}$. On the other hand, it is consistent that there is a non-principal ultrafilter $\mathcal{U}$ over ω such that every automorphism of $S_\mathcal{U}$ is inner. It is not known whether there always is a non-principal ultrafilter $\mathcal{U}$ such that $S_\mathcal{U}$ has non-inner automorphisms.