Research Statement for the Young Set Theory Workshop 2011

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My research interests focus on ultrafilters on semigroups $(S, \cdot)$, most prominently countable ones such as $(\omega, +)$, $(\omega, \cdot)$ and $([\omega]^{<\omega}, \cup)$. The field of algebra in the Stone-Čech compactification studies the extension of semigroup operations to βS; such extensions yield a semigroup operation on βS with idempotent elements and a minimal ideal both of which give rise to algebraic Ramsey-type theorems on the underlying semigroups as well as applications in topological dynamics. (Un)fortunately, the field developed almost without set theoretic "problems", i.e., almost all results are theorems of ZFC.

The classical ultrafilter notions (such as P-points and Q-points) on the other hand are extremely neutral with respect to the algebraic structure, e.g., no sum of two ultrafilters on ω can be a P- or Q-point. The set theoretic techniques related to these classical notions often turn out to be inadequate to attack questions about the ultrafilters relevant to the algebraic structure.

In my thesis my research focus lay on algebraic problems that had only been solved consistently, trying to indentify how set theoretic methods come into the picture. This led to results on union ultrafilters, on the existence of various types of idempotent ultrafilters in different models and on ultrafilters on $(\omega, +)$ with a maximal group $\cong \mathbb{Z}$ (and stronger properties).

During my current DFG fellowship I am working on the reverse situation, studying set theoretic means to attack open "algebraic" questions such as whether every continuous homomorphism $\beta\omega \to \omega^*$ is constant[1] or the existence of an infinite increasing chain of idempotent ultrafilters.[2] Approaching these problems with set theoretic machinery is difficult because the manipulation of idempotent ultrafilters, say, via forcing, has not really been studied before. I'm also interested in applications of idempotent ultrafilters in other set theoretic constructions, e.g., the Mildenberger-Shelah model for $NCF \not\Rightarrow FD$ or Blass's model for $Con(\mathfrak{u} < \mathfrak{g})$.

Finally, thanks to François Dorais I have recently developed an interest in the reverse mathematics of Neil Hindman's Finite Sums Theorem, a crucial tool in this field, as well as other open questions regarding its proofs.

[1] In the 1980s Dona Strauss proved such maps have a finite image.

[2] Here, chain means chain in the partial order of idempotent elements defined as $p \leq q$ iff $p \cdot q = q \cdot p = p$.