# Research Statement

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I have been working on constructions of rigid structures. A structure $\mathcal{A}$ is said to be *rigid* if its only automorphism is the identity. For instance, any finite linear ordering, or indeed any ordinal, is rigid. I also consider wider classes of maps on $\mathcal{A}$ than automorphisms, viz. embeddings, endomorphisms, epimorphisms, monomorphisms, bimorphisms (which are all various weakenings of the notion of automorphism). I have mainly considered dense linear orders without endpoints (though graphs are another case of interest) and given a number of constructions to illustrate what can happen.

A classical construction due to Dushnik and Miller [**?**] shows (using the Axiom of Choice) that $\mathbb{R}$ has a dense subset $X$ of cardinality $2^{\aleph_0}$ which is rigid with respect to automorphisms. It is easy to modify this example to make it also rigid with respect to embeddings and epimorphisms (the latter are maps $f : X \to X$ which is surjective and such that $x \leq y \Rightarrow f(x) \leq f(y)$). In [**?**] is shown how to find such an $X$ which is rigid for automorphisms but which admits *many* embeddings. A more precise question is to enquire what the possible *values* are of the embedding monoid $Emb(X, \leq)$ for an automorphism rigid $(X, \leq)$. For instance I can show that is can be isomorphic to $(\mathbb{N}, +)$, with a similar result for the epimorphism monoid $Epi(X, \leq)$.

For chains, only four of the possible six monoids are distinct, namely $Aut(X, \leq), Emb(X, \leq), Epi(X, \leq)$ and $End(X, \leq)$, and I can demonstrate all possible consistent combinations of equalities and inequalities between these. Work on the questions about what the moniods can actually be is still in progress.

# References

[1] DROSTE, M., TRUSS, J.K. *Rigid chains admitting many embeddings.* Proc. Amer. Math. Soc. **129** (2001), pp. 1601-1608.

[2] DUSHNIK, Ben and MILLER, E.W., *Concerning similarity transformations of linearly ordered sets.* Bull. Amer. Math. Soc, **46** (1940), pp. 322-326.