ON THE NOTION OF GUESSING MODEL

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My current research focuses on the notion of guessing model. This has been analyzed and introduced in [1]. The ultimate and most likely out of reach ambition in this work is to provide by means of guessing models useful tools to show that for a given model W of MM, $(\aleph_2)^W$ has an arbitrarily high degree of supercompactness in some simply definable inner model V.

A guessing model come in pair with an infinite cardinal δ:

- $\aleph_0$-guessing models provide an interesting characterization of all large cardinal axioms which can be described in terms of elementary embedding $j : V_\gamma \to V_\lambda$. In particular supercompactness, hugeness , and the axioms I_1 and I_3 can be characterized in terms of the existence of appropriate $\aleph_0$-guessing models.
- In a paper with Weiss [2] we showed that PFA implies that there are $\aleph_1$-guessing models, and that in many interesting models W of PFA such $\aleph_1$-guessing models M can be used to show that in some inner model V of W, $M \cap V$ is an $\aleph_0$-guessing models belonging to V and witnessing that $\aleph_2$ is supercompact in V.
- In [1] I also outline some interesting properties guessing models have in models of MM. For example assume θ is inaccessible in W, then:
  1. If W models PFA, then for a stationary set G of $\aleph_1$-guessing models $M \prec H_\theta$ the isomorphism-type of M is uniquely determined by the ordinal $M \cap \aleph_2$ and the order type of $M \cap Card$ where $Card$ is the set of cardinals in H_θ.
  2. In the seminal paper of Foreman Magidor and Shelah [4] on Martin's maximum and in a recent work by Sean Cox [3] several strong forms of diagonal reflections are obtained, for example Cox shows:

     > Assume MM holds in V. Then for every regular θ there is S stationary set of models $M \prec H_\theta$ such that every $M \in T$ computes correctly stationarity in the following sense:
     > *For every $X \in M$ and every set $R \in M$ subset of $[X]^{\aleph_0}$ if R is projectively stationary in V then R reflects on $[M \cap X]^{\aleph_0}$.*
  3. We can improve (1) and (2) above to further argue that in a model V of MM, $G \cap S$ is stationary.

Such results even if rather technical are attributing to $\aleph_2$ properties shared by supercompact cardinals in the sense that $\aleph_0$-guessing models M are characterized by property (1) when $\aleph_2$ is replaced by some suitable inaccessible cardinal $\kappa \in M$ and satisfy many strenghtenings of property (2).

References

[1] Matteo Viale, *On the notion of guessing model*, submitted, available at: `http://fiesh.homeip.net/guessing_model.pdf`, 2010, 17 pages.

[2] Matteo Viale, Christoph Weiss, *On the consistency strength of the proper forcing axiom*, submitted, available at: `http://fiesh.homeip.net/viale_weiss.pdf`, 2010, 20 pages.

[3] Sean Cox, *The diagonal reflection principle*, submitted, available at: `http://wwwmath.uni-muenster.de/logik/Personen/Cox/Research/DRP.pdf`, 2010, 11 pages.

[4] Matthew Foreman, Menachem Magidor, Saharon Shelah *Martin's maximum, saturated ideals, and non-regular ultrafilters. I*, Annals of Mathematics. Second Series, **127(1)** (1988), 1–47.