Research Statement

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I am a PhD student at Université Paris 7. At the moment I am mostly interested in questions about absoluteness of the following form: Suppose that $V \subseteq W$ are models of set theory with the same cardinal numbers and $V \models \exists x \phi(x)$. Can we find an object $A \in V$ satisfying $\phi(A)$ in both V and W?

In particular, I am interested in the question above in the case that κ is an uncountable cardinal and $\phi(A)$ states that A is a partition of κ into stationary sets. For example from [2] we know that ω_1 can be divided into $\aleph_0$ many stationary sets in V, all of which remain stationary in W. On the other hand, by [1], there is a forcing extension $V[G]$ of V which preserves ω_1 but no partition of ω_1 into $\aleph_1$ many stationary sets remains such in $V[G]$ (although greater cardinals might be collapsed.) To what extent can these results be generalized for $\kappa > \omega_1$?

These kind of questions are of particular interest when V and W model forcing axioms. For example, by results in [4], if both $V \subseteq W$ model PFA, and for every κ, the ω-cofinal ordinals below κ^+ can be partitioned into κ many stationary sets in a way described above, then $\mathrm{Ord}^\omega \cap V = \mathrm{Ord}^\omega \cap W$. It has been further conjectured in [3] that if $V \subseteq W$ both satisfy PFA, then $\mathrm{Ord}^{\omega_1} \cap V = \mathrm{Ord}^{\omega_1} \cap W$. The basic informal question is: to what extent does PFA (or MM) fix its models?

References

[1] Paul B. Larson: The nonstationary ideal in the $\mathbb{P}_{max}$ extension, The Journal of Symbolic Logic Volume 72, Number 1, 2007

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[3] Boban Veličković: Logic Colloquium 2006, Lect. Notes in Logic, Association for Symbolic Logic, Chicago, IL, (2009), pp. 328–360.

[4] Matteo Viale: The Proper Forcing Axiom and the Singular Cardinal Hypothesis, Journal of Symbolic Logic 71(2) (2006) 473-479