

RESEARCH STATEMENT

Edoardo Rivello, rivello.edoardo@gmail.com

I am interested in any set-theoretical topic that originates its motivation from a philosophical, foundational or metamathematical issue. I have just completed my Ph.D. with the thesis “Set Theory with a Truth Predicate”, under the supervision of Alessandro Andretta (Università di Torino, Italy). Currently I’m working, together with the University of Cambridge Set Theory Group, on models of the stratified fragments of Zermelo-Fraenkel Set Theory.

The notion of a stratified formula was introduced by Quine, in his “New Foundations” [Quine, 1937], as a device to avoid Russell’s paradox. He added to the axiom of Extensionality the Comprehension axiom-schema restricted to stratified formulæ, giving an axiomatization of Set Theory — known as NF, from Quine’s seminal paper — as an alternative to ZF. The problem of the consistency of NF relative to ZF is still open. Moreover there are also many other questions to answer about the subsystems of ZF that allow only stratified instances of the axiom-schemata of Separation and Replacement. Among these theories the most obvious to think of are: the stratified axioms of ZF within a stratified version of the axiom of Infinity, the stratified theorems of ZF, and the intersection of NF and ZF.

[Forster, 2004] introduced a class model of the stratified axioms of ZF — the class of the Hereditarily Symmetric sets (HS) — built up using a new notion of permutation of a model. The Cambridge Set Theory Group has started to explore further this idea to build other similar constructions. In particular I’m interested in various attempts to build analogues of the Gödel’s L using a stratified notion of definability or stratified operations. We know that Gödel’s two presentations of L and Jensen’s all give the same inner model of ZF. However, turning to the stratified analogues of these constructions, it is not clear if the equivalences still hold, neither, what axioms of ZF are true. Another related question is the relationship between these models and the class HS. I think that finding the right answers to these questions will help us to better understand the internal structure of L and, more in general, of similar Inner Model constructions.

References

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