RESEARCH STATEMENT FOR YOUNG SET THEORY, 2011

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Most of my research has focused on the following areas: Tukey theory of ultrafilters on ω, maximal almost disjoint families of sets and functions, preservation theorems for iterated forcing, and consistency with CH.

0.1. **Cofinal types of ultrafilters.** Given ultrafilters $\mathcal{U}$ and $\mathcal{V}$ on ω, we say that $\mathcal{V}$ is *Tukey reducible to* $\mathcal{U}$, and write $\mathcal{V} \leq_T \mathcal{U}$ if there is a map $\phi : \mathcal{U} \to \mathcal{V}$ such that $\forall a, b \in \mathcal{U}\,[a \subset b \implies \phi(a) \subset \phi(b)]$ and $\forall e \in \mathcal{V} \exists a \in \mathcal{U}\,[\phi(a) \subset e]$. We say $\mathcal{U}$ and $\mathcal{V}$ are *Tukey equivalent*, and write $\mathcal{U} \equiv_T \mathcal{V}$, if $\mathcal{V} \leq_T \mathcal{U}$ and $\mathcal{U} \leq_T \mathcal{V}$. There is a general notion of Tukey reducibility for arbitrary directed posets, of which this is a special case. Several general structure and nonstructure theorems are known regarding Tukey types of uncountable directed sets. In the case of ultrafilters, Tukey reducibility is a coarser notion of reducibility than the well studied Rudin-Keisler (RK) reducibility. In joint work with Todorčević, we consider the question of when Tukey reducibility is equivalent to RK reducibility. This is similar in spirit to asking when a automorphism of $\mathcal{P}(\omega)/\mathrm{FIN}$ is induced by a permutation of ω, which was a famous problem in the history of set theory. The most outstanding question regarding cofinal types of ultrafilters is the following long standing problem of Isbell.

Question 1. *Is it consistent that for every ultrafilter $\mathcal{U}$ on ω, there exists $\{x_\alpha : \alpha < \mathfrak{c}\} \subset \mathcal{U}$ such that for every $A \in [\mathfrak{c}]^\omega\, [\bigcap_{\alpha \in A} x_\alpha \notin \mathcal{U}]$?*

0.2. **Almost disjoint families.** We say that two infinite subsets a and b of ω are *almost disjoint or a.d.* if $a \cap b$ is finite. We say that a family $\mathscr{A}$ of infinite subsets of ω is *almost disjoint or a.d. in* $[\omega]^\omega$ if its members are pairwise almost disjoint. A *Maximal Almost Disjoint family, or MAD family in* $[\omega]^\omega$ is an infinite a.d. family in $[\omega]^\omega$ that is not properly contained in a larger a.d. family. Two functions f and g in ω^ω are said to be *almost disjoint or a.d.* if they agree in only finitely many places. We say that a family $\mathscr{A} \subset \omega^\omega$ is *a.d. in* ω^ω if its members are pairwise a.d., and we say that an a.d. family $\mathscr{A} \subset \omega^\omega$ is *MAD in* ω^ω if $\forall f \in \omega^\omega \exists h \in \mathscr{A}\,[|f \cap h| = \omega]$. We say that $p \subset \omega \times \omega$ is an *infinite partial function* if it is a function from some infinite subset $a \subset \omega$ to ω. An a.d. family $\mathscr{A} \subset \omega^\omega$ is said to be *Van Douwen* if for any infinite partial function p there is $h \in \mathscr{A}$ such that $|h \cap p| = \omega$. We answered an old question of Van Douwen by proving that Van Douwen families exist.

We have also answered a question of Shelah and Steprāns about almost disjoint families in $[\omega]^\omega$ that is closely related to the metrization problem for countable Fréchet groups. Let FIN denote the non-empty finite subsets of ω. Given an ideal $\mathcal{I}$ on ω, we say that $P \subset \mathrm{FIN}$ is *$\mathcal{I}$-positive* if $\forall a \in \mathcal{I} \exists s \in P\,[a \cap s = 0]$. Given an a.d. family $\mathscr{A} \subset [\omega]^\omega$, let $\mathcal{I}(\mathscr{A})$ denote the ideal on ω generated by $\mathscr{A}$. We say that an a.d. family $\mathscr{A} \subset [\omega]^\omega$ is *strongly separable* if for each $\mathcal{I}(\mathscr{A})$-positive $P \subset \mathrm{FIN}$, there is $a \in \mathscr{A}$ and $Q \in [P]^\omega$ such that $\bigcup Q \subset a$. Thus this notion is gotten from the well known notion of a completely separable a.d. family by replacing integers with finite sets in the definition. Shelah has recently proved that completely separable a.d. families exist if $\mathfrak{c} < \aleph_\omega$. But we show that strong separability behaves differently by proving that it is consistent that there are no strongly separable a.d. families and $\mathfrak{c} = \aleph_2$.

The following are some interesting open problems regarding almost disjoint families.

Question 2. *Is there a completely separable a.d. family?*

Question 3. *Is there an uncountable a.d. family $\mathscr{A} \subset [\omega]^\omega$ such that for every $\mathcal{I}(\mathscr{A})$-positive $P \subset \mathrm{FIN}$, there is a $Q \in [P]^\omega$ consisting of pairwise disjoint sets so that $\forall a \in \mathcal{I}(\mathscr{A})\,[|a \cap (\bigcup Q)| < \omega]$?*

Question 4. *Is there an Sacks indestructible MAD family?*

0.3. **Preservation theorems.** We have answered a question of Kellner and Shelah by proving the following: Let γ be a limit ordinal and let $\langle \mathbb{P}_\alpha, \mathring{\mathbb{Q}}_\alpha : \alpha \leq \gamma \rangle$ be a countable support (CS) iteration. Suppose that for each $\alpha < \gamma$, $\Vdash_\alpha$ "$\mathring{\mathbb{Q}}_\alpha$ is proper" and that $\mathbb{P}_\alpha$ does not turn $\mathbf{V} \cap \omega^\omega$ into a meager set. Then $\mathbb{P}_\gamma$ does not do so either.

Question 5. *Let γ be a limit ordinal and let $\langle \mathbb{P}_\alpha, \mathring{\mathbb{Q}}_\alpha : \alpha \leq \gamma \rangle$ be a CS iteration. Suppose that for each $\alpha < \gamma$, $\Vdash_\alpha$ "$\mathring{\mathbb{Q}}_\alpha$ is proper" and $\mathbb{P}_\alpha$ does not add a Cohen real. Is it true that $\mathbb{P}_\gamma$ also does not add a Cohen real?*

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