Research Statement

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March 17, 2011

My research centers around topics in descriptive set theory and the theory of iterated forcing on the one hand, and on forcing axioms and large cardinals on the other hand.

In my Ph.D. thesis [?], I investigated regularity properties of the two ideals $\mathcal{M}$ (meager) and $\mathcal{N}$ (null).

Theorem 1 ([?])**.** *Given the consistency strength of a Mahlo, there is a model where all projective sets are Lebesgue measurable but there is a Δ^1_3 set without the Baire property.*

In the next two years, I plan to do research in the area of cardinal characteristics on ω. Not assuming CH, it becomes an interesting field of investigation to obtain and study models where these characteristics have different cardinality.

If we strive to find models where $\mathfrak{c} = \omega_2$, many techniques are known for forcing different values for different characteristics (for example, [?] and [?]). Alternatively, if we want models for $\mathfrak{c} = \omega_3$ then virtually any question involving a relation between three characteristics (obeying the usual known restraints such as the ones in Cichońs diagram) is open, e.g.:

Question 1. *Construct a model where* $\mathrm{cov}(\mathcal{M}) = \omega_1$, $\mathfrak{d} = \omega_2$ *and* $\mathfrak{c} = \omega_3$.

I'm currently cooperating with Stefan Geschke to solve the following question:

Question 2. *Find a model where* $\mathfrak{c} = \omega_3$ *and* $\mathfrak{hm} = \omega_2$. *Possibly also consider* $\mathrm{cov}(\mathcal{M}) = \omega_1$.

One approach to this question would be to start with a carefully chosen model and add ω_3 many Cohen reals.

Lately Aspero and Mota [?] have found a way of iterating proper forcing of size ω_1 for length ω_3 using *elementary submodels as side conditions*, an idea which was introduced and investigated by Todorčević. Their work may offer a blue-print how to deal with other iterations in a slightly more general manner, in contrast to the ad-hoc approaches that have been employed in this field so far, possibly by allowing side conditions both of size ω_1 and of size ω as was done recently by Italy Neeman.

The axioms $\mathbf{OCA}_{[\mathrm{ARS}]}$ and $\mathbf{OCA}_{[\mathrm{T}]}$ demand that certain homogeneous sets for open colorings of certain Polish spaces exist. Both have numerous applications, and both are consequences of PFA (but not equiconsistent with PFA). In a spectacular result Moore [?] has shown that together, they imply $2^\omega = \omega_2$.

Similar approaches as the ones described above are also interesting for the following problem:

Question 3. *Is* $\mathbf{OCA}_{[\mathrm{ARS}]}$ *consistent with* $\mathrm{unif}(\mathcal{M}) = \omega_1$*?*

In my master's thesis I investigated the forcing axiom $\mathrm{FA}(\mathbf{\Sigma}^1_3, \Gamma)$, that is

$$\forall P \in \Gamma \quad V \prec_{\mathbf{\Sigma}^1_3} V^P,$$

for various classes of Γ, with respect to their consistency strength, e.g. obtaining:

Theorem 2 ([?], [?])**.** $\mathrm{FA}(\mathbf{\Sigma}^1_3, \mathrm{ccc})$ *together with "ω_1 is inaccessible to reals" is equiconsistent with the existence of a* lightface Σ^1_2-indescribable *cardinal.*

To see this, one associates an Aronszajn tree to a hypothetical failure of Σ^1_2 reflection in L. The rest of the argument combines a coding technique from [?] with an argument very similar to the classical one showing that the tree property holds at ω_2 under PFA.

Similar ideas are elaborated by Italy Neeman and Ernest Schimmerling (see especially [?]). In particular, Italy Neeman uses a morass-like construction to obtain a similar higher order reflection principle from PFA:

Theorem 3. *Assuming V is a proper forcing extension of an L-like model W and* PFA *holds. Then there is a Σ^2_1-indescribable 1-gap $[\kappa, \kappa^+]$ in W.*

The assumption of an L-like model was made plausible by recent work of Sy Friedman and Peter Holy [?], who showed that any model of set theory has an extension which is L-like (i.e. satisfies a strong form of condensation).

Unfortunately it is very unclear how to obtain larger gaps; obtaining Σ^2_1-indescribable gaps $[\kappa, \lambda]$ for every $\lambda \geq \kappa$ would yield $W \vDash \kappa$ is supercompact.

A different approach was taken by Viale and Weiß. In his thesis, Weiß has isolated a combinatorial property of supercompact cardinals, the *list property* or $\mathrm{ITP}(\kappa)$, which can be subtracted from inaccessibility, in the sense that $\mathrm{ITP}(\omega_2)$ can hold—and it does hold under PFA ([?], [?]).

On the other hand, as conjectured by Viale and Weiß, $\mathrm{ITP}(\omega_2)$ does not have as startling consequences as PFA. For example, in a joint paper with Shelah, it is proved that $\mathrm{ITP}(\omega_2)$ is consistent with arbitrarily large continuum (assuming a supercompact cardinal; see our forthcoming [?]). I also plan to generalize this work to larger cardinals and prove e.g. the relative consistency of $\mathrm{ITP}(\omega_3)$ and $2^{\omega_2} > \omega_3$.

Viale and Weiß show:

Theorem 4 ([?])**.** *If W is a proper forcing extension of V by a standard iteration of length κ, where κ is inaccessible and in V and $W \vDash$* PFA *and $\kappa = \omega_2$. Then κ is supercompact in V.*

Question 4. *Can you drop some of the assumptions from theorem 4?*

In [?], Bagaria strengthens a classical characterization of supercompactness in terms of higher order reflection, or Löweinheim Skolem type properties. In a joint project Bagaria and I plan to investigate if this characterization can be used to answer question 4.