RESEARCH PROPOSAL

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I am a second year MSc student at Eötvös Loránd University, Hungary. I am mainly interested in set theoretical topology. Recently, I have been investigating the connections between D-spaces and covering properties.

Definition 0.1 (E. van Douwen). *A space X is said to be a D-space (or has property D) iff for every open neighborhood assignment U, one can find a closed discrete $D \subseteq X$ such that $X = \bigcup_{d\in D} U(d) = \bigcup U[D]$.*

I recommend G. Gruenhage's survey on D-spaces [3], which summarizes the facts and the work done in the topic, stating numerous fascinating open problems. One of the main problems with D-spaces, is that we lack theorems stating that a classical covering property weaker than compactness implies property D. As Gruenhage says, "... it is not known if a very strong covering property such as hereditarily Lindelöf implies D, and yet for all we know it could be that a very weak covering property such as submetacompact or submetalindelöf implies D!"

In a joint work with Xu Yuming [7], we examined the D-property of some generalized metric spaces: generalized stratifiable spaces, elastic spaces and the Collins-Roscoe mechanism.

Investigating D-spaces, Arhangel'skii introduced the class of aD-spaces.

Definition 0.2 (Arhangel'skii, [1]). *A space X is said to be* aD *iff for each closed $F \subseteq X$ and for each open cover $\mathcal{U}$ of X there is a closed discrete $D \subseteq F$ and $N : D \to \mathcal{U}$ with $x \in N(x)$ such that $F \subseteq \bigcup N[D]$.*

Interestingly, aD-spaces are much more docile than D-spaces; Arhangel'skii showed, that every submetalindelöf space is aD [2]. Answering a question of Arhangel'skii [2], I proved that there exists an aD, non D-space [6]; the counterexamples use Shelah's club guessing theory. Nevertheless, the questions about main covering properties and D-spaces remain open.

In [5], I answered questions raised by Guo and Junnila [4] concerning characterization of linearly D-spaces; that is, in the definition of D-spaces, we only consider monotone neighborhood assignments. Also, I proved that the existence of certain "locally nice" aD, non D-spaces is independent.

Now, I am interested in getting a better insight on non D-spaces, which are linearly D and aD. I hope, that this will shed some light on the question, whether every Lindelöf space is D.

References

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