Research Statement

Alexander Primavesi

I began my doctoral research by looking at a long-standing question of the set theorist I. Juhasz, of whether the axiom $\clubsuit$ implies the existence of a Suslin tree (an uncountable tree with no uncountable chains or antichains). The existence of Suslin trees is known to be independent of ZFC. $\clubsuit$ is one of a family of axioms known as 'guessing axioms' and is a natural weakening of $\diamondsuit$, which is itself a strengthening of the Continuum Hypothesis and is known to imply the existence of a Suslin tree. $\clubsuit + \mathrm{CH}$ is equivalent to $\diamondsuit$, so $\clubsuit$ can be thought of as $\diamondsuit$ without the cardinal arithmetic assumptions. Juhasz's question is one of a class of natural questions that ask: how different are $\diamondsuit$ and $\clubsuit$?

My research is concerned with several such questions. Two examples are the following:

- $\diamondsuit$ has an *invariance property* in the sense that making small changes to its definition won't, in general, get you a different (either strictly stronger or strictly weaker) statement. To what extent is this invariance property shared by $\clubsuit$?
- It is known that $\clubsuit$ is consistent with $\neg$CH, but there are related questions that remain unanswered. For example, is it possible to force $\clubsuit$ from a model of $\neg$CH without collapsing 2^ω?

These are questions that came to light in my research into Juhasz's question and could have application in answering it. The definition of $\clubsuit$ is as follows:

Definition 0.1. ($\clubsuit$) *There is a sequence $\langle A_\delta : \delta \in Lim(\omega_1)\rangle$ such that $A_\delta \subseteq \delta$ and $\sup(A_\delta) = \delta$, and if $X \subseteq \omega_1$ is uncountable then the set $\{\delta < \omega_1 : A_\delta \subseteq X\}$ is stationary.*