Borel equivalence relations and the conjugacy problem

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One of the most interesting classification problems in mathematics is the *conjugacy problem* for a given group. Indeed, when studying a finite group it is usual to write down its class equation and study the set of conjugacy classes. For larger groups (say Polish groups) one similarly should study the conjugacy equivalence relation.

The idea of this talk is to isolate the complexity of the conjugacy relation on a few special groups. The relevant notion of complexity comes from an area of study called Borel equivalence relations. Here, if E, F are equivalence relations on (standard) Borel spaces X, Y, we say that E is *Borel reducible* to F iff there exists a Borel function $f\colon X \to Y$ such that

$$x \; E \; x' \iff f(x) \; F \; f(x') \; .$$

I will give a short introduction to the theory of Borel reducibility, and then use this tool to analyze the conjugacy problem for some of the most famous and well-studied groups in logic: the automorphism groups of $\mathbb{Q}$, the random graph, and other ultrahomogeneous structures.