Young set theory workshop

Seminarzentrum Raach near Vienna, 15 - 19 February 2010

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Contents

Welcome

I am pleased to welcome you all to Young Set Theory 2010!

This is the third annual Young Set Theory workshop and thanks in large part to the enthusiasm and cooperation of the participants, this workshop series has firmly established itself as a fixture in the conference circuit. However, it is becoming clear that in many aspects, Young Set Theory is a larger and useful concept which should be discussed and formalised. The two main aspects of this are questions of organisation and questions of mathematics.

Organisatorially, Young Set Theory has the potential to become, in a more formalised way, the research, learning and career network that was intended in the founding of it. While the workshops themselves continue to be extremely successful in uniting young researchers throughout the world and in various set-theoretic disciplines, there is a clear possibility to sustain this level of communication and to give the nascent network a more permanent form, practical for day-to-day work. We need a central place to collect and share information. Collections of lecture notes and annotations on papers and books, conference and job announcements could be included as well as a database of research statements so that we can get to know each other professionally, not just at these workshops. In another direction we could form working groups and fora, to better coordinate communication.

Mathematically, set theory needs directions. We need to have goals to work towards and an understanding of the larger framework in which to put our work. Mathematics has always thrived on the interplay between fields and here we have an enormous freedom: set theory can be applied to nearly every area of mathematics. Also, in the words of M. Goldstern, set theory is the study of infinity and in this sense it is also an extremely broad and diverse field in itself. It has its place in mathematics as a pure subject as well.

If we convince others that our work is relevant, we will have the power to help decide the future of mathematics and logic. One of the major ways to achieve this is to connect with other mathematicians. This should always be viewed as a two-way street. We have backgrounds in general mathematics and a deep knowledge of set theoretic methods. It is the nature of a cooperation that neither party is an expert in the other's field; nevertheless, the transposition of a set theoretic technique to a new field has helped settle difficult problems. The main problem here is that of language - we need to come to a common understanding despite our differences in standardised terminology.

This language barrier is also the major problem for other mathematicians to understand set theory. Apart from the basic books on set theory (Jech, Kunen, etc.) there is surprisingly little literature (books, research papers or survey articles) which makes modern set theory accessible to general mathematicians or even students in the field. We need to remind ourselves and others about what we are doing, what we have achieved in the context of our framework - our models of set theory. This helps, not only to help other set-theorists and mathematicians use our results and to train the next generation of set theorists, but also to keep our field thriving.

Successful mathematicians have at their disposal a large repertoire of techniques and results, often from different areas of mathematics. The more we study these diverse areas, the more connections we will find.

It is not for me to say what are the major questions or goals of set theory. My hope is that here at this meeting, we can start this discussion. I ask that you bring your ideas and enthusiasm, a willingness to help make these goals a reality. Together we can move set theory forward in exciting ways and in turn, make a positive impact on mathematics.

I wish you all thought-provoking, enlightening and rewarding days at this conference.

Katie Thompson
on behalf of the organisers

Organising Committee:

Bernhard Irrgang, Bonn
David Schrittesser, Vienna
Katie Thompson, Vienna

Scientific Committee:

Piotr Borodulin-Nadzieja, Wroclaw
Andrew Brooke-Taylor (chair), Bristol
Vera Fischer, Vienna
Gunter Fuchs, Staten Island/CUNY
Asger Törnquist, Vienna
Matteo Viale, Torino

Tutorial Speakers

Uri Abraham, Beer-Sheva
Greg Hjorth, Melbourne
Justin Moore, Cornell
Ralf Schindler, Münster

Postdoc Speakers

Inessa Epstein, Cal Tech
Thomas Johnstone, CUNY and Vienna
Bart Kastermans, Boulder
Wieslaw Kubis, Warsaw
Philipp Schlicht, Bonn
Lyubomyr Zdomskyy, Vienna

Acknowledgements

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Additionally, the organisers would like to thank Ioanna Dimitriou for the wonderful website and booklet cover design; Ajdin Halilović, Ekaterina Neugodova and Marcin Sabok for their organisational support.

1 Schedule

Schedule Young Set Theory Workshop 2010

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 – 9:50 10:00 – 10:50	Tutorial: Ralf Schindler	Tutorial: Uri Abraham	Tutorial: Uri Abraham	Tutorial: Justin Moore	Tutorial: Justin Moore
10:50 – 11:10	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:10 – 12:00	Bart Kastermans	Tutorial: Ralf Schindler	Lyubomyr Zdomskyy	Tutorial: Greg Hjorth	Tutorial: Greg Hjorth
12:10 – 13:00	Inessa Epstein		Lunch + EXCURSION (Raxalpe)		
13:00 – 14:30	Lunch	Lunch		Lunch	Lunch
14:30 – 17:30	Discussions	Discussions		Discussions	Discussions
	with coffee at 16:00	with coffee at 16:00		with coffee at 16:00	with coffee at 16:00
17:30 – 18:20	Thomas Johnstone	Philipp Schlicht		Wiesław Kubiś	Leave for Vienna
19:00 - ?	Dinner	Dinner	Dinner	Dinner	Dinner in Vienna 20:00

2 Practical information

Registration fee
The registration fee includes accommodation (including Friday 19 February), meals Monday - Friday including the social dinner, bus to and from Raach and the use of all facilites at the Seminarzentrum Raach. Not included are drinks (other than water, which is excellent and comes directly from the mountains) and the excursion.

Bus to and from Raach
Bus to Raach leaves from the KGRC at 20:00 on Sunday, 14 February
Bus from Raach leaves at 17:30 on Friday, 19 February

Excursion
On Wednesday 17 February at 12:00, we will take a bus to the Raxseilbahn, which is a cable car up a mountain. The bus ride should take 10-15 minutes. The cable car ascends to 1545m in 8 minutes. At the top, we will eat lunch and there will be a possibility to rent snowshoes (weather permitting).

Group picture
The group picture will take place Thursday 18 February just before lunch (13:00).

Lodging in Vienna for 19 February
Participants: Wombats City Hostel Vienna "The Lounge"
Mariahilfer Straße 137, A-1150 Vienna, Austria
http://www.wombats-hostels.com/vienna/the-lounge
Breakfast is included, but not towels. Please bring your own!

Speakers: Arcotel Boltzmann
Boltzmanngasse 8
1090 Vienna, Austria
http://www.arcotel.at/boltzmann
Breakfast is not included.

Social dinner
Pizzeria Il Sestante
Piaristengasse 50, 1080 Vienna
20:00, Friday 19 February

3 Abstracts of tutorials and post-doc talks

Uri Abraham
Some classical c.c.c forcings

For those who are already familiar with the consistency proof of Martin's Axiom, the next step is probably some finite support iteration of posets that are c.c.c but not so easily obtained. 1) A model of Baumgartner and Shelah in which there is a thin-tall boolean algebra (of hight omega two). 2) A model in which every function from R to R is monotonic on an uncountable set, 3) Baumgartner's celebrated model in which every two aleph one dense sets of reals are isomorphic.

Inessa Epstein
Descriptive set theory and measure preserving group actions

We are going to consider the space of free, measure preserving, ergodic actions of a countable group on a standard probability space. This space is of high importance in functional analysis. We will consider equivalence relations on this space - in particular, the equivalence relation given by orbit equivalence - and discuss the Bore complexity of the equivalence relation. Two group actions are orbit equivalent if these is a measurable way of amost everywhere identifying the orbits given by the actions. Descriptive set theoretic aspects that are of interest will be introduced as well as recent results concerning these group actions.

Greg Hjorth
Effective Descriptive Set Theory and Admissible Sets

In these talks I will discuss a circle of ideas which are represent vital techniques in the theory of light faced Σ^1_1, Π^1_1, and Δ^1_1 sets but whose representation in the existing literature of set theory is obscure.

One of the motivating themes in modern descriptive set theory is that insight into certain complexity classes can be achieved by analyzing the appropriate inner model. For instance it follows from Shoenfield absoluteness that a set $A \subset \omega$ is Σ^1_2 if and only if it is Σ_1 definable over Gödel's constructible universe L. For certain purposes, constructibility gives us the right way of thinking about Σ^1_2.

In this sequence of lectures we will be looking at sets far closer to ground.

Definition 3.1 ω_1^{ck} *is the supremum of the recursive ordinals – where we an ordinal* α *is said to be recursive if there is a recursive well order of* ω *with order type* α*.*

It turns out that the study of $L_{\omega_1^{\mathrm{ck}}}$ presents us with a model for understanding Π^1_1.

Theorem 3.2 ω_1^{ck} *is the least ordinal* $\alpha > \omega$ *such that* L_α *is admissible.*

I will attempt to sketch a proof of this using an effective version of the Kunen-Martin theorem.

Theorem 3.3 *(Spector-Gandy) Let* A *be a subset of* ω*. Then* A *is* Π^1_1 *if and only if it is* Σ_1 *definable over* $L_{\omega_1^{\mathrm{ck}}}$*.*

If time and interest allows, I also hope to show how some of the relevant technology can be used to give a proof of Lusin-Novikov:

Theorem 3.4 *(Lusin-Novikov) The image of a countable to one Borel function is Borel.*

Further topics might include Gandy-Harrington forcing and its application in proving dichotomy theorems for Borel equivalence relations.

Thomas Johnstone
The Resurrection Axioms

I will discuss a new class of forcing axioms, the Resurrection Axioms (RA), and the Weak Resurrection Axioms (wRA). While Cohen's method of forcing has been designed to change truths of the set-theoretic universe you live in, the point of Resurrection is that some truths that have been destroyed by forcing can in fact be resurrected, i.e. forced to hold again. In this talk, I will illustrate how RA and wRA are tied to bounded forcing axioms such as MA and BPFA, and how they affect the size of the continuum. The main theorem will show that RA and many instances of wRA are equiconsistent with the existence of an uplifting cardinal, a large cardinal notion consistent with V=L. This is joint work with Joel Hamkins.

Bart Kastermans
Formalizing Set Theory

We all know the vast difference between the notion of proof as described in an introductory logic class, and the one we use on a daily basis when doing mathematics. The simple notion of proof has a large benefit that it is easy to check. Ideally all our proofs would be given with this much detail (next to the intuitive explanation) so that a higher degree of certainty can be obtained. In practice this is undoable, or at least highly unpleasant, to do by hand. To facilitate this certain computer systems have been developed to help with this. In this talk I'll give a short introduction to such a system, and show how I have used it to formalize a simple example of a cofinitary group.

Wiesław Kubiś

Applications of elementary submodels to Banach spaces

We shall describe how to use elementary substructures of $\langle H(\theta), \in\rangle$ for constructing projections in Banach spaces. Given a Banach space $E \in H(\theta)$ satisfying certain conditions, an elementary submodel of $\langle H(\theta), \in\rangle$ induces a norm one linear projection from E onto the closure of $E \cap M$. Having this property, one can build a transfinite sequence of projections, called a *projectional resolution of the identity.* This further implies some geometric and structural properties of the Banach space, e.g. the existence of a strictly convex renorming, the Lindelöf property of its weak topology and so on.

We shall also discuss the use of elementary substructures for studying compact spaces and their Banach spaces of continuous functions. Given a non-metrizable compact space K, an elementary submodel M of a suitably large structure $\langle H(\theta), \in\rangle$ induces a quotient map $q^M\colon K \to K/M$, where K/M is obtained from K by identifying points that are not separated by any function from $C(K) \cap M$. Analyzing quotient maps of the form q^M one can usually say something about the compact K. For instance, Bandlow's result [1] states that K is Corson compact if and only if for every countable M, q^M is one-to-one on the closure of $K \cap M$.

The talk is partially based on [4, 7, 10].

[1] I. Bandlow, *A characterization of Corson-compact spaces*, Comment. Math. Univ. Carolinae **32** (1991) 545–550

[2] W. Kubiś, *Compact spaces generated by retractions*, Topology Appl. **153** (2006) 3383–3396

[3] W. Kubiś, *Banach spaces with projectional skeletons*, J. Math. Anal. Appl. **350** (2009) 758–776

[4] W. Kubiś, H. Michalewski, *Small Valdivia compact spaces*, Topology Appl. **153** (2006) 2560–2573

Justin Moore
The Proper Forcing Axiom

In this tutorial, I will give an introduction to the Proper Forcing Axiom, starting with the definition of properness. An emphasis will be placed on how to construct a proper partial order and how to verify its properness. I will also discuss two consequences of PFA — the Open Coloring Axiom and the P-ideal Dichotomy — which have proved very useful in applications.

Ralf Schindler
Mice and forcing absolutenenss

I will discuss the concept of a mouse and give examples of mice. I will show that any real can be made generic over mice with Woodin cardinals for the extender algebra with ω generators and that any subset of ω_1 can be made generic over mice with a measurable Woodin cardinal δ for the extender algebra with δ generators. This will be done in the first part. In the second part I will show how to use the results from the first part to prove versions of $L(R)$- or Σ^2_1-absoluteness with respect to forcing.

Philipp Schlicht
Descriptive set theory at uncountable cardinals

There are several analogies to classical results in descriptive set theory for the spaces ${}^\kappa\kappa$, where κ is a regular uncountable cardinal with $\kappa^{<\kappa} = \kappa$. I will speak about regularity properties and definable equivalence relations on ${}^\kappa\kappa$. The perfect set property for analytic sets holds in the model where an inaccessible is collapsed to κ^+ by $< \kappa$-closed forcing. In this context there is a counterexample to Silver's theorem: a Δ^1_1 equivalence relation with κ^+ many equivalence classes, but no perfect set of inequivalent elements of ${}^\kappa\kappa$. However, a weaker form of Silver's theorem is consistent.

Lyubomyr Zdomskyy
Projective mad families

The main purpose of the talk will be to analyze how low in the projective hierarchy one can consistently find a mad subfamily of $[\omega]^\omega$ or ω^ω. Below is a brief discussion of the results we are going to present. We are also going to discuss some open problems in this area.

A classical result of Mathias [4] states that there exists no Σ^1_1 definable mad family of infinite subsets of ω. One of the two main results of [2] states that there is no Σ^1_1 definable ω-mad family of functions from ω to ω. By [5, Theorem 8.23], in L there exists a mad subfamily of $[\omega]^\omega$ which is Π^1_1 definable. Moreover, $V = L$ implies the existence of a Π^1_1 definable ω-mad subfamily A of ω^ω, and hence a Π^1_1 definable ω-mad subfamily A of $[\omega]^\omega$, see [2, 6].

Regarding the models of $\neg$CH, it is known that ω-mad subfamilies of $[\omega]^\omega$ remain so after adding any number of Cohen subsets, see [3] and references therein. By the results of Raghavan [6], we conclude that the ground model ω-mad families of functions remain so in forcing extensions by countable support iterations of a wide family of posets including Sacks and Miller forcings. A straighforward argument based on the Shoenfield's Absoluteness Theorem gives that if a ground model Π^1_1 definable mad family *remains mad* in a forcing extension, it remains Π^1_1 definable by means of the same formula. From the above it follows that the Π^1_1 definable ω-mad family in L of functions constructed in [2] remains Π^1_1 definable and ω-mad in $L[G]$, where G is a generic over L for, e.g., the countable support iteration of Miller forcing of length ω_2.

In all models of $\neg$CH mentioned above the bounding number $\mathfrak{b}$ equals ω_1. It is easy to see that the ground model ω-mad families are not anymore mad in extensions by posets adding dominating reals. Therefore some other approach is needed in order to obtain projective mad families in models of $\mathfrak{b} > \omega_1$. Using almost disjoint coding one can prove [3] the consistency of the existence of a Π^1_2 definable ω-mad family of infinite subsets of ω (resp. functions from ω to ω) together with $\mathfrak{b} = 2^\omega = \omega_2$.

This is a joint work with Sy-David Friedman.

[1] Friedman, S.D., Zdomskyy, L, *Projective mad families,* submitted.

[2] Kastermans, B., Steprāns, J., Zhang, Y., *Analytic and coanalytic families of almost disjoint functions,* Journal of Symbolic Logic **73** (2008), 1158–1172.

[3] Kurilić, M.S., *Cohen-stable families of subsets of integers*, Journal of Symbolic Logic **66** (2001), 257–270.

[4] Mathias, A.R.D., *Happy families,* Annals of Mathematical Logic **12** (1977), 59–111.

[5] Miller, A.W., *Infinite combinatorics and definability,* Annals of Pure and Applied Logic **41** (1989), 179–203.

[6] Raghavan, D., *Maximal almost disjoint families of functions,* Fundamenta Mathematicae **204** (2009), 241-282.

4 Research statements

Uri Abraham
Ben-Gurion University of the Negev, Israel

I am interested in combinatorial questions that deal with the infinite, and especially those that involve consistency results. For example, with James Cummings, we studied polychromatic Ramsey theory. I am also interested in application of the notion of model to questions of modeling concurrency.

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Dominik Adolf
University of Münster, Germany

I'm interested in a diverse array of set-theoretic subjects, including large cardinals, forcing and generic ultrapowers. My Research though has a clear focus on inner model theory.

One of the main techniques I'm working with is the core model induction. Like most of inner model theory it is used to find lower bounds on consistency strength. The most result obtained by using the core model induction is the closure of the universe under $M_n^\#$ for every natural number n, i.e. there exists for every set X a mouse built over X and containing n woodin-cardinals bigger than the rank of X. To obtain this we utilize the K-existence dichotomy, which broadly states, that, assuming the closure of the universe under $M_n^\#$ for some n, either the universe is closed under $M_{n+1}^\#$ or for some set X the core model over X exists.

This first step was already applied to a pcf-theoretic problem in [1]. I intend to expand upon this result utilizing an approach similar to the one in the recent paper [2], which shall yield the mouse capturing condition W_α^* for all ordinals α. W_α^* states, that for any set of reals $U \in J_\alpha(\mathbb{R})$, such that both U and it's complement admit scales in $J_\alpha(\mathbb{R})$, any real x and any natural number n, there exists a mouse containing x and n woodins that "captures" U. It is an elementary fact, that W_α^* for all α yields $AD^{L(\mathbb{R})}$. I intend to strengthen this, building on methods of G. Sargsyan and R. Ketchersid (see [4]), reaching a model M of AD, such that Θ^M is a regular limit of it's Solovaysequence.

Furthermore I'm interested in any application of the stacking mice method introduced here: [3]

[1] M. Gitik, S. Shelah, R. Schinlder, *Pcf theory and Woodin cardinals*, Lecture Notes in Logic **27** (2006) 172 – 205

[2] J. Steel, S. Zoble, *Determinancy from strong reflection*, preprint.

[3] R. Jensen, E. Schimmerling, R. Schinlder, J. Steel, *Stacking Mice*, Journal of Symbolic Logic **74** (2009) 315–335.

[4] G. Sargsyan, *A tale of hybrid mice*, PHD Thesis.

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Carolin Antos-Kuby
University of Vienna, Austria

I am a student at the Kurt Gödel Research Center currently writing my master thesis (Diplomarbeit) under the supervision of Sy D. Friedman. My thesis deals with the question of forcing extensions which do not create new large cardinals, i.e. whether a cardinals κ, which has a certain large cardinal property in the forcing extension $V^{\mathbb{P}}$, has the same large cardinal property in the ground model V. I'm referring mainly to the work of Joel David Hamkins.
One of the first results concerning this question is a theorem by Levy and Solovay, which states that for small forcings (i.e. a forcing notion P that has size less than a cardinal κ) κ is measurable in the forcing extension V^P if and only if it is measurable in the ground model V. This can be extended to several large cardinal notions (for example [1]).
Concentrating on the direction from the extension to the ground model, Hamkins generalizes this result to all forcing extensions where the forcing notion has a closure point at δ (i.e. it factors as $\mathbb{P}*\dot{\mathbb{Q}}$, where $\mathbb{P}$ is nontrivial, $|\mathbb{P}|\delta$ and $\Vdash_{\mathbb{P}} \dot{\mathbb{Q}}$ is δ-strategically closed). This includes for example the Silver iteration, the canonical forcing of the GCH, the Laver preparation, the lottery preparation and reverse Easton iterations. For such extensions every suitably closed embedding $j : \bar{V} \to \bar{N}$ in the extension $\bar{V}$ lifts an embedding $j \restriction V : V \to N$ amenable to the ground model. This can of course be applied to all large cardinal properties which are witnessed by such kinds of embeddings, for example measurable, supercompact, huge, strong and Woodin cardinals. (see [2])
I'm currently working on the last part of the thesis which is about a Theorem by Laver, stating that if V is a model of ZFC, $\mathbb{P} \in V$, and $V[G]$ is a $\mathbb{P}$-generic extension of V, then in $V[G]$, V is definalbe from parameters $V_{\delta+1}$, for $\delta = |\mathbb{P}|^+$. Laver proves this by using Hamkins' results from above (see [3]).

[1] J. Hamkins, H. Woodin *Small forcing creates neither strong nor Woodin cardinals*, Proc. Amer. Math. Soc. 128 (2000) 3025–3029.

[2] J. Hamkins *Extensions with the approximation and cover properties have no new large cardinals*, Fund. Math. 180 (2003) 237–277

[3] R. Laver *Certain very large cardinals are not created in small forcing extensions*, Annals of Pure and Applied Logic 149 (2007) 1–6

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Dana Bartosova
Charles University Prague, Czech Republic

I am interested in interactions between topology, set theory and model theory. At the moment, I am trying to find analogies between direct and projective Fraïssé theory based on the papers by Kechris, Pestov and Todorčević [1] and Irwin and Solecki [2]. The former paper reveals connections between the direct Fraïssé theory, Ramsey theory and topological dynamics and the latter paper introduces the dual notion of the projective Fraïssé theory. However very few projective Fraïssé classes has been described so far and the question remains whether one can find analogical results to those for the direct Fraïssé theory.

I am still interested in applications of the model-theoretic notion of elementarity to the theory of compact Hausdorff spaces and reflecting properties of non-metric compacta to metric compacta.

[1] A.S. Kechris, V.G. Pestov, S. Todorčević *Fraiïssé limits, Ramsey theory, and topological dynamics of automorphism groups*, GAFA Geom. funct. anal. **15** (2005) 106–189.

[2] T.L. Irwin, S. Solecki *Projective Fraïssé limits and the preudo-arc*,Trans. Amer. Math. Soc. **358** (2006), 3077–3096.

[3] R. Camerlo *Characterising quotients of projective Fraiïssé limits*, preprint.

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Rafael Benjumea
Instituto Venezolano de Investigaciones Cientificas, Venezuela

The main subjects that I am interested in are descriptive set theory and Ramsey theory.

Currently I am trying to establish a dichotomy theorem that allow classify definable graphs according to if its Borel chromatic number is finite or infinite. Kechris, Solecki and Todorcevic [1] have shown that for any analytic graph G on a Polish space X, exactly one of the following holds:

1. The Borel chromatic number of G, $\chi_B(G)$, is less or equal than $\aleph_0$.

2. Exists a continuous homomorphism of G_0 into G.

Where G_0 is an analytic graph with uncountable Borel chromatic number.

The first step to try to generate an analogous theorem in the case finite vs infinite, is a complete study of the shift graph over $[\mathbb{N}]^{\mathbb{N}}$, the collection of all infinite sets of natural numbers, this graph has Borel chromatic $\aleph_0$, and seems like a possible candidate in order to classify some interesting subclass of analytic graphs. Is important to note that even in the simplest case, when the graph G is generated by a Borel countable-to-1 function, not much is known.

Furthermore, we are interested in study another class of "definable" chromatic numbers on analytics graphs, like measurables and Baire chromatics numbers.

[1] A. S. Kechris, S. Solecki, and S. Todorcevic, *Borel Chromatic numbers*, Advances in Mathematics, **141**(1999), n,1 : 1-44.

[2] B. Miller, *Measurable chromatic numbers*, The Journal of Symbolic Logic. **73**(2008) 1139-1157.

[3] C. A. Di Prisco, and S. Todorcevic, *Borel chromatic numbers of shift graphs*, in preparation.

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Tristan Bice
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My research focuses on set theoretic aspects of Hilbert spaces. In my masters thesis I looked at generalizing the notion of analytic subset to subspaces of a Hilbert space while now, in the 1st year of my PhD, I have been looking at cardinal invariants defined from projections on a Hilbert space.

Linearly Analytic Subspaces If V is a vector space then the projection of a subspace of $V \oplus V$ will be a subspace of V. In particular, I looked at what I termed 'linearly analytic subspaces', subspaces of a separable Hilbert space H that are the projection of a closed subspace of $H \oplus H$. As suggested by my terminolgy, I believe that such subspaces are the linear analogs of analytic subsets of a Polish space X which, recall, are the projections of a closed subset of $X \times X$. Indeed, just as analytic subsets have a number of different characterizations, so too do linearly analytic subspaces. For example, just as analytic subsets are the image of continuous maps on ω^ω, linearly analytic subspaces are the image of continuous linear maps on l^2. Analogous theorems about them can also be proved. For example, just as all analytic subsets are countable or have cardinality or $\mathfrak{c}$, linearly analytic subspaces are finite dimensional or have Hamel dimension $\mathfrak{c}$. Likewise, just as analytic subsets are closed under finite (or even countable) unions and intersections, linearly analytic subspaces are closed under finite sums and intersections. I only developed the basic theory of these linearly analytic subspaces and I firmly believe this could be taken much further.

Cardinal Invariants from Projections modulo Compact Operators We can view l^2 as a 'quantum' analog of ω. Then subspaces of l^2 or, equivalently, (orthogonal) projections onto subspaces of l^2 become the quantum analogs of subsets of ω. In this context 'modulo compact operators on l^2' becomes the natural analog of 'modulo finite subsets of ω'. More specifically, for projections P and Q on l^2 we define

$$P \leq^* Q \Leftrightarrow P - PQ \text{ is compact.}$$

From this we can define cardinal invariants from $(\mathcal{P}(\mathcal{B}(l^2)), \leq^*)$ ($\mathcal{P}(\mathcal{B}(l^2))$ is the collection of projections on l^2) in analogy to the way classical cardinal invariants were defined from $(P(\omega), \subseteq^*)$.

There are a few complications, however. For one thing, a particular cardinal invariant will often have different analogs depending on whether we use orthogonal complementation ($P^\perp = 1 - P$) or arbitrary complementation. For example, we have the following analogs of splitting.

$$\begin{aligned} P \text{ weakly splits } Q \quad &\Leftrightarrow \quad P \wedge^* Q \neq 0 \text{ and } Q \nleq^* P. \\ P \text{ strongly splits } Q \quad &\Leftrightarrow \quad P \wedge^* Q \neq 0 \text{ and } P^\perp \wedge^* Q \neq 0. \end{aligned}$$

This naturally leads to a weak splitting number $\mathfrak{s}^*$ and a strong splitting number $\mathfrak{s}^\perp$, the minimum cardinality of a weakly or strongly splitting family respectively. Another complication is that (the $=^*$-equivalence classes of) $(\mathcal{P}(\mathcal{B}(l^2)), \leq^*)$ is not a lattice. This makes defining some things a little tricky like, for example, ultrafilters and, consequently, the analog of the ultrafilter number $\mathfrak{u}$. However, I was at least able to characterize exactly when projections P and Q have a g.l.b. and l.u.b. by looking at the spectral family, or the essential spectrum, of the self-adjoint operator PQP, and this may be relevant to defining such cardinal invariants.

In any case, many ZFC inequalites that can be proved between the classical cardinal invariants can also be proved, albeit with more effort, between their quantum analogs. The relationship between each classical cardinal invariant and its quantum analog seems to be more difficult to pin down, except for the quantum analogs of $\mathfrak{b}$ and $\mathfrak{d}$, which are known to equal their classical counterparts. Presumably (and hopefully) this is not the case for all the invariants. Using the fact that projections onto block subspaces are dense in $(\mathcal{P}(\mathcal{B}(l^2)), \leq^*)$ I have, however, been able to bound some quantum cardinal invariants by similar invariants related to interval partitions of ω, for example $\mathfrak{s}^\perp \leq \mathfrak{s}^{\mathrm{IP}}$ where $\mathfrak{s}^{\mathrm{IP}}$ is the finite splitting number in [3]. These in turn, can sometimes be related to the classical cardinal invariants, for example, in [3] it is shown that $\mathfrak{s}^{\mathrm{IP}}$ is actually just the maximum of $\mathfrak{b}$ and $\mathfrak{s}$. Recently, in an effort to try and prove an inequality between the quantum and interval partition analogs of the groupwise density number $\mathfrak{g}$, namely $\mathfrak{g}^{\perp} \leq \mathfrak{g}^{\mathrm{IP}}$, I was lead to the interesting problem of generalizing Talagrand's characterization of meagre subsets of ${}^{\omega}2$ (as given in [1] 5.2) to $\mathcal{P}(\mathcal{B}(l^2))$, which has been done for certain meagre subsets in [5] chapter 3. Finally, I have obtained a few consistency results, extending similar results in [4], but most of the interesting consistency questions remain open and will be the focus of my research from now on.

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I am interested in applications of set theory to measure theory, topology and functional analysis. Below I list my current areas of interest.

Efimov and Grothendieck spaces in forcing extensions. Efimov space is a compact infinite space without converging sequences and without a copy of $\beta\omega$. It is not known if such spaces exist in ZFC. Recently Dow and Fremlin ([4]) showed that a Efimov space exists in standard random model and Brech ([3]) showed the existence of a space with even stronger properties (Grothendieck space of weight $\omega_1 < \mathfrak{c}$) in the model obtained by the product of ω_2 Sacks forcings. There are plenty of questions which can be considered here (apart of the question of the existence of Efimov spaces in ZFC). Among other questions, I would like to investigate if there is a Grothendieck space in the random model.

Separable measures. I am interested in the question about a characterization (combinatorial, topological) of the class of Boolean algebras admitting only separable measures. In [1] I showed that every Boolean algebra either admits a uniformly regular measure or it carries a measure which is non-separable and that the class of minimally generated Boolean algebras is a (quite rich) class of Boolean algebras which carry only separable measures. I'm interested also in a similar question about a characterization of Boolean algebras supporting a (strictly positive) separable measure (MRP(separable) in terms of [5]). Recently, together with Mirna Dzamonja, we found a characterization of Boolean algebras carrying strictly positive uniformly regular measures.

Cardinal invariants of density filter. Cardinal invariants of density ideal have been profoundly investigated by Fremlin, Hrusak & Hernandez, Soukup & Farkas (see eg [7], [6]) and other mathematicians. However, there are still some natural open questions in this subject. Some of them, motivated by problems from Banach space theory, are formulated in [2]: e.g. if the minimal cardinality of a family with s.f.i.p. without any condenser (a pseudo-intersection in the sense of the density filter; confront [2] for the precise definition) can be consistently greater that the pseudo-intersection number $\mathfrak{p}$.

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My research in set theory centres around large cardinal axioms and class forcing. Lately my work has focused on a large cardinal axiom known as Vopěnka's Principle. This axiom lies in consistency strength beyond supercompact cardinals, but is weaker than the existence of almost huge cardinals. It has found applications in category theory and algebraic topology, and I have been working with Joan Bagaria of the University of Barcelona studying these applications. I have also been writing up results about the indestructibility of Vopěnka's Principle to certain class forcings — if it held before the forcing, it will still hold after the forcing. This contrasts with the usual situation, in which large cardinals only become indestructible after some preparatory forcing.

I'm also interested in morasses, rank-to-rank embeddings, and models of $\neg$AC, amongst other things.

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My concern is with mesurable functions from the Baire space in itself. My purpose is to find out how to define a hierarchy of Borel functions, and what it looks like. The general idea is to find well-suited games such that a Borel function can be seen as a strategy in those games if and only if the degree of inverse images of open sets is bounded in the Borel hierarchy.

For example, the case of continuous functions as already been done:
Let $\mathcal{N}$ be the Baire space of infinite sequences of integers, along with the product topology, which admits as basis the sets $N_s := \{x \in \mathcal{N} : s \text{ is a prefix of } x\}$, where s is a finite sequence of integers.
Let now $f : \mathcal{N} \to \mathcal{N}$ be a function, and G_f a two-player, perfect information game in which, at turn $i \in \mathbb{N}$, player I chooses an integer x_i, and player II a finite sequence of integers u_i such that if $i < j$ then $u_i \subseteq u_j$. Let then $x = (x_i)_{i\in\mathbb{N}}$ be the infinite sequence of I's moves, and $y = \bigcup_{i\in\mathbb{N}} u_i$ be the union of II's finite sequences, then the winning condition is that player II wins G_f if and only if y is in $\mathcal{N}$ and $f(x) = y$.
We can now give the following result:

A function $f : \mathcal{N} \to \mathcal{N}$ is continuous iff player II as a winnning strategy in G_f.

If we now allow player II to erase, we can obtain all functions of the first Baire class, but is it possible to find a game for each Baire class? Is it moreover possible to refine the Baire hierarchy of functions, in order to find, by means of games, partition theorems, as the Jayne-Rodgers theorem? Those are examples of questions I am working on.

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I'm working on the Katowice problem, i.e. is it consistent with ZFC that $P(\omega)/fin$ is isomorphic with $P(\omega_1)/fin$?

Recently I was able to construct a model, in which $d = \omega_1$ and there is a C-tower. $\mathcal{T} = \{T_\alpha : \alpha \in \omega_1\}$ is an (increasing) C-tower iff $\mathcal{T}$ generates a non-meager ideal $<\mathcal{T}>$, $T_{\alpha+1} \setminus T_\alpha$ is a Strong-Q-sequence and $\mathcal{T}$ has nonempty intersection with each nonprincipal p-ultrafilter. The next step could be to verify whether this result could be expanded so that $P(\omega)/<\mathcal{T}>$ is a σ-complete algebra.

The result was achieved by iteration of Gregorieff-like forcing notions. The same tools can be used to simplify a forcing construction from [4].

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My research is on the interaction of large cardinals and forcing.

It is well known that the failure of GCH at a measurable cardinal is equiconsistent with the existence of a measurable cardinal κ of Mitchell order κ^{++}. I have proven the following results similar in spirit to the above. First, the failure of GCH at κ where κ is κ^+-supercompact is equiconsistent with the existence of a κ^+-supercompact cardinal. In other words, no additional large cardinal hypothesis is needed to force the GCH to fail at a κ^+-supercompact cardinal. Tallness is a large cardinal concept first used by Woodin and Gitik, and studied in their own right by Hamkins in [2]. A cardinal κ is *θ-tall* if there is an elementary embedding $j : V \to M$ with critical point κ such that $j(\kappa) > \theta$ and $M^\kappa \subseteq M$. We say that κ is *tall* if κ is θ-tall for every θ. The second result is that the failure of the GCH at λ where κ is λ-supercompact is equiconsistent with the existence of a λ-supercompact cardinal κ that is also λ^{++}-tall.

I am also interested in indestructibility results for strongly compact cardinals. Apter and Gitik [3] proved that, in a forcing extension, a supercompact cardinal can become strongly compact and the least measurable while being fully indestructible by $< \kappa$-directed closed forcing. In [4], Hamkins showed that one can eliminate the supercompactness requirement so that a strongly compact cardinal κ (with no extra assumptions) can be made indestructible by Cohen forcing $\mathrm{Add}(\kappa, 1)$. I would like to address questions such as, "how much indestructibility is possible for a strongly compact cardinal κ when κ is not fully supercompact?"

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I am interested in large cardinals, inner model theory, and forcing. I am particularly interested in properties of countably complete ideals—e.g. the nonstationary ideal on ω_1—and how these ideals are related to large cardinals in inner models.

A classic example of such a property is precipitousness: an ideal I is precipitous iff whenever G is generic for the boolean algebra $P(A)/I$, then $ult(V, G)$ is wellfounded; here A is the fixed collection whose subsets are being measured by I. If there is a precipitous ideal on ω_1 (i.e. where $A = \omega_1$) then there is an inner model with a measurable cardinal, and in fact a measurable cardinal is the optimal large cardinal assumption for obtaining a precipitous ideal. Moreover, the existence of a precipitous ideal on κ implies weak covering holds at κ for K, where K is the core model below a Woodin cardinal (this is recent work of Schindler).

However, I am mainly interested in natural properties of ideals which do not imply precipitousness, yet still have high consistency strength. My recent research—joint with Martin Zeman—has focused on such questions.

I also continue to work on covering arguments, particularly extending parts of Mitchell's Covering Theorems to the core model below a Woodin cardinal (e.g. proving that if $cf^V(\gamma) < |\gamma|^V$, $\gamma > \omega_2$, and γ is regular in K then γ is measurable in K). This also involves finding a relationship between the nonstationary ideal and large cardinals in K.

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I am interested in some of the largest large cardinal hypotheses that are currently not known to be inconsistent. These include the axiom I0 which was first used by Woodin in proving that determinacy axioms follow from strong enough large cardinal axioms. The axiom I0 states that for some λ there is a (non-trivial) elementary embedding

$$j : L(V_{\lambda+1}) \to L(V_{\lambda+1})$$

with critical point less than λ. This axiom sits just below the inconsistent axiom that there exists an elementary embedding

$$j : V_{\lambda+2} \to V_{\lambda+2},$$

which was proved inconsistent by Kunen. The proof also shows that there cannot be an elementary embedding $j : V \to V$ assuming AC. Therefore, if I0 is consistent then we cannot have that $L(V_{\lambda+1})$ satisfies choice.

The generalizations of I0 are usually of the form there exists an elementary embedding $j : L(N) \to L(N)$ with critical point less than λ and with $V_{\lambda+1} \subseteq N \subseteq V_{\lambda+2}$. For instance, suppose that $X \subseteq V_{\lambda+1}$ then such an axiom postulates the existence of an elementary embedding from $L(X, V_{\lambda+1})$ to itself. The axiom that asserts the existence of what is called a proper elementary embedding from $L(X, V_{\lambda+1})$ to itself turns out to have implications very similar to the consequences of assuming AD in $L(\mathbb{R})$. Specifically, if Θ is defined analogously to how Θ is defined for models of determinacy (i.e. as the sup of ordinals which are surjective images of $V_{\lambda+1}$ in $L(V_{\lambda+1})$) then Θ is the limit of cardinals which are weakly inaccessible and limits of measurable cardinals witnessed by the club filter on a stationary set, along with other analogous consequences. Therefore it appears as though there is a deep structural connection between these two hypotheses, though it is not clear to what extent this analogy holds.

The main reference for this material is Hugh Woodin's Suitable Extender Sequences which is in progress.

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I am primarily concerned with the study of measure-preserving actions of countable groups. The study of measure-preserving actions of countable groups on standard probability spaces up to orbit equivalence was initiated by Dye in the 1950's. Since then, the subject has become an important meeting point of ergodic theory, operator algebras and Borel equivalence relations.

Let (X, μ) be a standard Borel space with a non-atomic probability measure (this is isomorphic to the interval $[0, 1]$ with the Lebesgue measure) and Γ be a countable group acting on (X, μ) by measure preserving transformations. This gives rise to the orbit equivalence relation E_Γ given by the orbits of the action. Two such actions $\Gamma \curvearrowright (X, \mu), \Delta \curvearrowright (Y, \nu)$ are *orbit equivalent* if there is a measurable bijection up to null sets identifying the orbits under Γ and Δ. The study of orbit equivalence rigidity is concerned with the rigid nature of Γ and determining which group theoretic properties of Γ may be recovered from the orbit equivalence relation E_Γ. The theory of orbit equivalence has strong connections with operator algebras. Orbit equivalence first appeared in a paper by Murray and von Neumann [7] via the "group measure space" construction. One may from a measure preserving free ergodic action of an infinite countable group obtain a type II_1 von Neumann factor with an abelian Cartan subalgebra. Two von Neumann algebras obtained in this way are isomorphic via an isomorphism preserving the Cartan subalgebras if and only if the corresponding actions are orbit equivalent.

We are primarily concerned with free, measure preserving an ergodic actions. The actions of two groups Γ and Δ on (X, μ) and (Y, ν) are orbit equivalent are orbit equivalent if there is a measurable bijection ϕ between A and B, which are conull invariant subsets of X and Y, respectively, such that for $x \in A$, $\phi(\Gamma \cdot x) = \Delta \cdot \phi(x)$. While amenable groups admit only one free, measure preserving, ergodic action up to orbit equivalence, the situation is quite different for non-amenable groups. In the past 10 years, the work of Hjorth [4], Gaboriau and Popa [3], Ioana [5], Gaboriau and Lyons [2] and Epstein [1] showed that actually every countable non-amenable group admits continuum many free, measure preserving ergodic actions.

With this result, one may fix a group Γ and consider the space of free, measure preserving, ergodic actions of Γ on a standard probability space. Then the equivalence relation OE_Γ given by the orbit equivalence of such actions may be studied in a descriptive set theoretic context. Tornquist [8] initiated the study of the Borel complexity of OE_Γ for certain classes of non-amenable groups by showing that for $n \geq 2$, $OE_{\mathbf{F}_n}$ is not Borel reducible to identity on the real and also that OE_Γ is not classifiable by countable structures for non-amenable Γ with property (T). Following the completion of the theorem that non-amenable groups admit continuum many orbit inequivalent actions, the results of Epstein, Ioana, Kechris and Tsankov [6] showed that OE_Γ is not classifiable by countable structures for any non-amenable Γ. In particular, it is not possible to assign a real-valued invariant to the equivalence classes of OE_Γ. Tornquist [9] also showed that for Γ with Kazhdan's property (T), isomorphism on torsion free abelian groups Borel reduces to OE_Γ; this, in particular, leads to the result that OE_Γ is not Borel for these kinds of groups. Much still remains open and to be explored concerning the Borel complexity of OE_Γ.

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My research in set theory is focused on combinatorics and cardinal invariants of analytic P-ideals, their connection with the classical cardinal invariants of the continuum, associated Borel relations, Borel Galois-Tukey connections between these relations, and related bounding and dominating properties of forcing notions.

An ideal $\mathcal{I}$ on ω is *analytic* if $\mathcal{I} \subseteq \mathcal{P}(\omega) \simeq 2^\omega$ is an analytic set in the usual product (polish space) topology of the Cantor-set. $\mathcal{I}$ is a *P-ideal* if for each countable $\mathcal{C} \subseteq \mathcal{I}$ there is an $A \in \mathcal{I}$ such that $I \subseteq^* A$ for each $I \in \mathcal{C}$, where $A \subseteq^* B$ iff $A \setminus B$ is finite. $\mathcal{I}$ is *tall* if each infinite subset of ω contains an infinite element of $\mathcal{I}$. For example, $\mathsf{fin} = [\omega]^{<\omega}$, $\mathcal{Z} = \{A \subseteq \omega : \lim_{n\to\infty} \frac{|A\cap n|}{n} = 0\}$, and $\mathcal{I}_{\frac{1}{n}} = \{A \subseteq \omega : \sum_{n\in A} \frac{1}{n+1} < \infty\}$ are analytic P-ideals, $\mathcal{Z}$ and $\mathcal{I}_{\frac{1}{n}}$ are tall as well.

Slawomir Solecki proved the following very useful characterization theorem (see [2]): Let $\mathcal{I}$ be an ideal on ω. Then $\mathcal{I}$ is an analytic P-ideal or $\mathcal{I} = \mathcal{P}(\omega)$ if and only if $\mathcal{I} = \mathrm{Exh}(\varphi) = \{A \subseteq \omega : \lim_{n\to\infty} \varphi(A \setminus n)\}$ for some lower semicontinuous submeasure φ on ω. Therefore each analytic P-ideal is $F_{\sigma\delta}$ (i.e. $\mathbf{\Pi}^0_3$).

Currently I am working on I-Luzin sets and related topics. Let I be an ideal on the set X. A set $S \subseteq X$ is (κ, λ)-*I-Luzin* if $|S| \geq \kappa$ and $|S \cap A| < \lambda$ for each $A \in I$.

A set $H \subseteq X$ is *I-accessible* (resp. *I-inaccessible*) if there is (resp. no) an $\subseteq$-increasing sequence $\langle A_\xi : \xi < \mu \rangle$ in I such that $H = \bigcup_{\xi<\mu} A_\xi$.

Let $\mathcal{I}$ be a tall ideal on ω. A sequence $\langle A_\alpha : \alpha < \kappa \rangle$ in $[\omega]^\omega$ is a *tower* if it is $\subseteq^*$-descending and it is not *diagonalizable*, i.e. it has no *pseudointersection*, that is a set $X \in [\omega]^\omega$ such that $X \subseteq^* A_\alpha$ for each $\alpha < \kappa$. Assume $\mathcal{F}$ is a filter on ω. A tower $\langle A_\alpha : \alpha < \kappa \rangle$ is a *tower in* $\mathcal{F}$ if $A_\alpha \in \mathcal{F}$ for each $\alpha < \kappa$.

If $\mathcal{I}$ is a tall ideal on ω, then let $\widehat{\mathcal{I}}$ be the ideal on $[\omega]^\omega$ generated by the sets in the form $\widehat{A} = \{X \in [\omega]^\omega : |X \cap A| = \omega\}$ for $A \in \mathcal{I}$. Clearly, $\mathcal{I}$ is a P-ideal iff $\widehat{\mathcal{I}}$ is a σ-ideal.

Let $\mathcal{I}$ be a tall ideal on ω. It is easy to check that if there exists a tower in the dual filter of $\mathcal{I}$, in $\mathcal{I}^*$, then $[\omega]^\omega$ is $\widehat{\mathcal{I}}$-accessible, and this implies that there are no (κ^+, κ)-$\widehat{\mathcal{I}}$-Luzin sets. It is unclear if the $\widehat{\mathcal{I}}$-accessibility of $[\omega]^\omega$ implies the existence of towers in $\mathcal{I}^*$.

Lajos Soukup and I proved that after adding ω_1 Cohen reals there are towers is $\mathcal{I}^*$ for each tall analytic P-ideal $\mathcal{I}$ (see [1]). Jörg Brendle proved that it is consistent with ZFC that there are no towers in the dual filters of tall analytic P-ideals (unpublished).

Problem: Is it consistent with ZFC that there is an (ω_2, ω_1)-$\widehat{\mathcal{I}}$-Luzin set for some tall analytic P-ideal $\mathcal{I}$?

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My main interests are in infinitary combinatorics, forcing and cardinal characteristics of the continuum. However, I have also interests in definability, as well as applications of set theoretic techniques to analysis and topology.

In the last few years, I have been working on obtaining various consistency results, requiring continuum greater than or equal to $\aleph_3$. In between those are the consistencies of $\mathfrak{b} = \mathfrak{a} = \kappa < \mathfrak{s} = \lambda$ as well as $\mathfrak{b} = \kappa < \mathfrak{s} = \mathfrak{a} = \lambda$, where $\kappa < \lambda$ are arbitrary regular uncountable cardinals (see [2], [9], [4], [10]). This lead to the development and use of interesting iteration techniques, which can be further extended to provide models in which three cardinal characteristics of the continuum have distinct values. Of interest remain the following questions: Is $\mathfrak{b} < \mathfrak{a} < \mathfrak{s}$ relatively consistent with the usual axioms of set theory? Is $\mathfrak{b} < \mathfrak{s} < \mathfrak{a}$ relatively consistent with the usual axioms of set theory? It might be expected that the techniques of [2], [10] can be also extended to provide the existence of some special maximal almost disjoint families, some special ultrafilters, as well as the consistency of $\mathfrak{b} = \kappa < \mathfrak{s} = \mathfrak{a} = \lambda$ without the assumption of a measurable cardinal. I have further interests in non-linear iterations, in particular template forcing ([3], [13]), combinatorics of uncountable cardinals and in some questions concerning large cardinals and forcing.

Apart from that, I am interested in forcing and definability, and the existence of some naturally definable combinatorial objects on the real line ([6], [7], [11]). In [2] it is shown that BPFA is consistent with the existence of a projective well-order of the reals. In [6], using the countable support iteration of S-proper posets, we introduce a gentle iteration techniques, which allows one to force the existence of a projective well-order on the reals and simultaneously control the values on some of the cardinal characteristics of the real line. There are natural continuations of this work in the context of measure and category ([7]).

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My research centers around ultrafilters on the natural numbers. More generally, I am interested in infinitary combinatorics, cardinal characteristics of continuum and applications of set theory in topology and algebra.

The key notion for my research has been the concept of an $\mathcal{I}$-*ultrafilter* (and two weaker notions defined analogously) which was introduced by Baumgartner [1]: Let $\mathcal{I}$ be a family of subsets of a given set X, such that $\mathcal{I}$ contains all singletons in X and is closed under subsets. A free ultrafilter $\mathcal{U}$ on ω is called an $\mathcal{I}$-ultrafilter, if for every mapping $f : \omega \to X$ there exists a set $U \in \mathcal{U}$ such that $f[U] \in \mathcal{I}$ (in the definition of weak $\mathcal{I}$-ultrafilters resp. $\mathcal{I}$-friendly ultrafilters only finite-to-one resp. one-to-one functions are considered). I have mainly studied $\mathcal{I}$-ultrafilters and both derived notions in the setting $X = \omega$ and $\mathcal{I}$ is an ideal on ω.

Most of the problems I am interested in come from the following two areas:

Existence and generic existence of $\mathcal{I}$-ultrafilters

I have proved in [3] that there exists in ZFC an $\mathcal{I}_{1/n}$-friendly ultrafilter for the summable ideal $\mathcal{I}_{1/n} = \{A \subseteq \omega : \sum_{n\in A} \frac{1}{n} < \infty\}$. There are two basic directions in which the result might be strengthened and a significant progress has not been done in either of them yet.

• Do $\mathcal{I}$-ultrafilters exist in ZFC for $\mathcal{I} = \mathcal{I}_{1/n}$? Or for some other analytic ideal $\mathcal{I}$?

• Is there an $\mathcal{I}_g$-friendly ultrafilter for every tall summable ideal $\mathcal{I}_g$? For $\mathcal{I}_{1/\sqrt{n}}$?

We say that a class of ultrafilters exists generically if every filter base of size less than $\mathfrak{c}$ can be extended to an ultrafilter in the class. Some results concerning generic existence of certain $\mathcal{I}$-ultrafilters were obtained by Brendle [2]. In joint work with Joerg Brendle we investigate the generic existence of other $\mathcal{I}$-ultrafilters. We have proved some consistency results involving cardinal characteristics of continuum and expect more.

Sums and products of ultrafilters

For a large class of ideals the product of $\mathcal{I}$-ultrafilters is again an $\mathcal{I}$-ultrafilter, for other this is not true. I would like to focus on some other ideals (e.g. van der Waerden ideal) for which the question has not been solved yet.

I want to understand better the connection between Q-points and rapid ultrafilters because this might be useful for the eventual construction of a model where rapid ultrafilters exist, but Q-points do not. It is known that the class of rapid ultrafilters is closed under products whereas the class of Q-points is not (see Miller's paper [4]).

• What is the smallest class of ultrafilters closed under products that contains all Q-points?

• Is it consistent with ZFC (or provable in ZFC) that this class of ultrafilters is strictly smaller than the class of all rapid ultrafilters?

[1] J. Baumgartner, *Ultrafilters on ω*, J. Symbolic Logic **60** (1995) 624–639.

[2] J. Brendle, *Between P-points and nowhere dense ultrafilters*, Israel J. Math. **113** (1999) 205–230.

[3] J. Flašková, *More than a 0-point*, Comment. Math. Univ. Carolin. **47** (2006) 617 – 621.

[4] A. W. Miller, *There are no Q-points in Laver's model for the Borel conjecture*, Proc. Amer. Math. Soc. **78** (1980) 103–106.

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My main interest in set theory is the study of the Forcing Axioms. There are several questions concerning this domain that I find very interesting; one of these is the research of a minimal principle deciding the size of the continuum. Very recently, David Asperó and Miguel Angel Mota have founded a new result which is strictly related to this question. By using a new iteration, they have constructed a model of ZFC in which the Weak Club Guessing fails and the size of the continuum is as large as one wishes. Maybe their technique can be generalized to produce models of other consequences of PFA in which the continuum is large. Now, consider the Proper Forcing Axiom PFA restricted to proper forcings of size $\aleph_1$, that will be hereafter denoted by $PFA(\aleph_1)$. It is an open question whether $PFA(\aleph_1)$ decides the size of the continuum and I think this is a crucial question.

Another subject that I found very interesting is the study of the reflection principles. There is, in particular, a nice principle known as the Semistationary Reflection Principle $SSRP$ which is equivalent to the statement that every ω_1-stationary preserving forcing is semiproper. Hiroshi Sakai proved that $SSRP$ is strictly weaker than the Reflection Principle. There is a lot of questions concerning this principle which are still open. For instance, the question of whether SRP implies the Singular Cardinal Hypothesis, or the Square Principle.

Finally, the problem of understanding the exact cardinal strength of each forcing axiom is very fascinating in my opinion. Any conjecture formulated in order to approach an answer to this question is captivating for me. One should ask, for example, if the Bounded Martin Maximum BMM implies that $L(\mathcal{P}(\omega_1)) \models NS_{\omega_1}$ is saturated. A positive answer would give an important information about the exact cardinal strength of BMM.

[1] U. Abraham. Proper Forcing to appear in *Handbook of Set Theory* Foreman, Kanamori, Magidor eds.

[2] J. Bagaria. Bounded forcing axioms as principles of generic absoluteness. *Archive for Mathematical Logic* (2000) 39: 393–401.

[3] H. Sakai. Semistationary and Stationary reflection. *Science in China Series A: Mathematics* eds. *Springer-Verlag* (2007) 50(5): 615–627.

[4] S. Shelah. Proper and Improper forcing. *Springer*. (1998).

[5] B. Veličković. Forcing Axioms and Cardinal Arithmetic. *Proceedings of the Logic Colloquium 2006*, Lecture notes in Logic, vol. 32. B. Cooper, H. Guevers, A. Pillay and J. Väänänen, eds., Association for Symbolic Logic, (2009), pp. 328–360.

[6] H. Woodin. *The axiom of determinacy, forcing axioms and the nonstationary ideal.* Walter de Gruyter (1999).

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I am interested in higher recursion theory, abstract set theory, descriptive set theory and in the application of set theory to questions in model theory, computability theory and proof theory. My earliest work was an application of Jensen's Square Principle and Silver's work on the Singular Cardinal Hypothesis to study Post's problem in higher recursion theory. A few years later I became interested in the application of forcing and infinitary model theory to the study of admissible ordinals, second order arithmetic and Scott ranks. I then began an extensive investigation of Jensen's coding method, which resulted in my book (Fine structure and class forcing, de Gruyter, 2000). Using coding, other methods of class forcing and Kleene's Recursion Theorem, I refuted Solovay's Pi-1-2 singleton conjecture, proved Solovay's admissibility spectrum conjecture and answered Jensen's question regarding the existence of reals which are minimal but not set-generic over L. In later work on coding (Genericity and large cardinals, Journal of Mathematical Logic, Vol. 5, No. 2, pp. 149–166, 2005) I produced reals which are class-generic but not set-generic and also preserve Woodin cardinals.

More recently I have been looking at connections between set theory and model theory, as well as several new programmes within set theory itself. In work with Hyttinen and Rautila (Classification theory and 0-sharp, Journal of Symbolic Logic, Vol. 68, No. 2, pp. 580–588, 2003) I showed that a first order theory is classifiable in the model-theoretic sense exactly if its models are classifiable in a sense that arises naturally in set theory. The paper "Internal consistency and the inner model hypothesis" (Bulletin of Symbolic Logic, Vol.12, No.4, December

2006, pp. 591–600) introduces the internal consistency programme, which aims to build inner models witnessing the consistency of set-theoretic statements, and which has also led to strong absoluteness principles, the most important of which is the Strong Inner Model Hypothesis (SIMH). The aim of the outer model programme (see "Large cardinals and L-like universes", in Set theory: recent trends and applications, Quaderni di Matematica, vol. 17, pp. 93–110, 2007) is to create Gödel-like models for large cardinal axioms. A third programme, closely related to the internal consistency programme, aims to prove the consistency for large cardinals of a wide range of combinatorial properties known to be consistent for small cardinals. A key advance on this latter programme was made in "Perfect trees and elementary embeddings" (Journal of Symbolic Logic, vol. 73, no. 3, pp. 906–918, 2008, joint with Katie Thompson), which introduced perfect set forcing into the large cardinal context, reproving old results with easier proofs as well as establishing many new results. An example of the latter is my joint work with Magidor (The number of normal measures, Journal of Symbolic Logic, Vol.74, No.3, pp. 1060 – 1080, 2009) concerning the possible number of normal measures on a measurable cardinal.

Most recently, I have turned to descriptive set theory. My work with Motto Ros (Analytic equivalence relations and bi-embeddability, http://www.logic.univie.ac.at/ sdf/papers/) shows that any analytic pre-order is Borel equivalent to an embeddability relation. With Fokina and Trnquist I have developed the (overlooked) effective theory of Borel equivalence relations, with some unexpected results.

Some other ongoing projects concern cardinal characteristics in the presence of projective wellorders (with Fischer, Zdomskyy), isomorphism relations on computable structures (with Fokina), the set theory of abstract elementary classes (with Koerwien), the descriptive set theory of finite structures (with Buss, Flum and Mller), condensation and large cardinals (with Holy), the tree property (with Halilovic) and definable failures of the singular cardinal hypothesis (with Honzik).

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Most of the classical combinatorial results of Ramsey type can be proved using Generic Absoluteness arguments. The traditional proofs of classical results, e.g., the Galvin-Prikry or Silver's theorems, hinge on a careful analysis of Borel or analytic partitions. But if one wants to generalize these results to more complex partitions, then one not only needs to assume some extra set-theoretic hypothesis – such as large cardinals, determinacy, or forcing axioms –, but the proof itself needs to be adapted accordingly. The advantage of using generic absoluteness is that the same proof for the Borel case generalizes readily to more complex partitions, under the additional hypothesis that the universe is sufficiently absolute with respect to its forcing extensions by some suitable forcing notions. In the case of the Ramsey property for sets of reals, the associated forcing notion is Mathias' forcing. In the case of perfect set properties, such as the Bernstein property, the associated forcing notions are Sacks forcing and its Amoeba. Typically, to each kind of partition property there are associated two forcing notions: P and Amoeba-P, so that assuming a sufficient degree of generic absoluteness under forcing with them, one can prove the desired Ramsey-type results. The combinatorial core of the problem turns out to be the following: prove that every element of the generic object added by Amoeba-P is P-generic.

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My research centers around applications of pure set theory and descriptive set theory in real analysis. In my doctoral thesis I study descriptive complexity of sets which appear naturally in real analysis. I show that set of strictly singular autohomeomorphisms of the unit interval is Π^1_1–complete, in particular non–Borel [5]. I investigate descriptive complexity of compact subsets of real line with prescribed Lebesgue density and porosity [4], [7], the set of all functions differentiable on co-countable sets in $C[0,1]$ [5], and the family of density preserving autohomeomorphisms [8]. I am also interested in studying of properties of small sets in Polish spaces [1], [2], [3].

[1] M. Balcerzak, S. Głąb, *On the Laczkovich–Komjath property of sigma-ideals*, Topology Appl., **157** (2010), 319-326.

[2] M. Balcerzak, S. Głąb, *Measure-category properties of Borel plane sets and Borel functions of two variables*, Acta Math. Hungar., to appear.

[3] P. Borodulin-Nadzieja, S. Głąb, *Ideals with bases of unbounded Borel hierarchy*, preprint.

[4] S. Głąb, *Descriptive properties related to porosity and density for compact sets on the real line*, Acta Math. Hungar., **116** (1-2) (2007), 61-71.

[5] S. Głąb, *Descriptive properties of families of autohomeomorphisms of the unit interval*, J. Math. Anal. Appl. **343** (2008) 835-841.

[6] S. Głąb, *On complexity of continuous functions differentiable on cocountable sets*, Real Anal. Exchange Vol. **34**(2), 2008/2009, pp. 1-9.

[7] S. Głąb, *Descriptive set-theoretical properties of an abstract density operator*, Cent. Eur. J. Math., **7**(4), 2009, 732-740.

[8] S. Głąb, F. Strobin *Descriptive properties of density preserving autohomeomorphisms of the unit interval*, preprint.

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I study Mathematics (5th year) and Informatics (3rd year) at Warsaw University. I am mainly interested in descriptive set theory, automata theory and applications of general topology, infinite combinatorics and game theory in these areas. These interests are reflected in my choice of courses I have attended during the past few years. Some more detailed examples include:

- almost disjoint families of subsets of ω, maximal almost disjoint families (MADs) of subsets of the Baire space ω^ω and their descriptive properties
- descriptive complexity of sets in the Cantor space, game theory applications
- automata on finite and infinite words over finite alphabet and their representations in MSO logic
- ultrafilters with their applications in various areas

Currently, I prepare to write my master's thesis on MAD families (mentioned above). I try to widen my knowledge of listed topics; in the future I plan to focus mainly on areas lying (more or less) in the intersection of them.

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I am interested in set theory, and its applications in the study of cardinal transfer principles.

At the moment I am in KGRC as a guest and I am working with Prof. Friedman on the following problems:

1. **The relation between gap-n-cardinal transfer principles**. The problem is to show that gap-$(n+1)$-cardinal transfer principle does not follow from gap-n-cardinal transfer principle.

2. **The effect of adding a real to models of set theory**. The main problem is the following question of Shelah and Woodin: Is it possible to force the total failure of GCH by adding a real?

I am also very interested in the Singular Cardinals Problem, and in particular in the global behaviour of the power set function. The following question is of great interest for me:

Question. Fix a natural number $m > 1$. For each limit ordinal α including 0, let $\varphi_\alpha : \omega \to \omega$ be an incresing function such that $\varphi_\alpha(0) = m$, and for each $n \epsilon \omega$, $\varphi_\alpha(n) > n$. Is there a model of set theory in which for each limit ordinal α, and each natural number n, $2^{\aleph_{\alpha+n}} = \aleph_{\alpha+\varphi_\alpha(n)}$.

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I am interested mainly in algebraic logic and model theory (especially stability theory) and their connections with finite combinatorics and complexity theory.

Currently I am working on finite extensions of Morley's categoricity theorem. Morley's theorem states that a (countable, first order) theory T is $\aleph_1$-categorical if and only if it is κ-categorical for all uncountable cardinal κ. As an extension, T is said to be *categorical in the finite* if, counting up to isomorphisms, every large enough finite subsets of T can have at most one n-element model for all $n \in \omega$. An easy example is the theory of algebraically closed fields. My aim is to find sufficient and necessary conditions for $\aleph_1$-categorical theories to be categorical in the finite.

Concerning algebraic logic I work on a question of Németi and Maddux. The problem is to find the "weakest strong" logic. One measure of "strength" of a logic is whether Gödel's incompleteness property holds for it. Another measure is the number of variables it uses. The fewer variables it has, the "weaker" our logic is. It was proved by Németi, improving a result of Tarski, that the logical counterpart of CA_3, i.e. first order logic using three variables has Gödel's incompleteness property (both syntactic and semantic). In fact, set theory can be built up using only three variables. A consequence of this is that the one-generated free three dimensional cylindric algebra $\mathsf{Fr}_1\mathsf{CA}_3$ is not atomic. It is also known that first order logic with two variables (the corresponding logic of CA_2) is decidable thus it does not enjoy Gödel's incompleteness property (not even the weak one). Recently I gave proofs for the corresponding logics of PA_3 and SCA_3, i.e. first order logic using three variables without equality but with permutations or substitutions of variables. Thus there remained the following open questions:

(1) Is it true that the one-generated free three dimensional diagonal-free algebra $\mathsf{Fr}_1\mathsf{Df}_3$ is atomic?

(2) Does incompleteness property hold for the logic of Df_3?

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I am interested in large cardinals and I have recently been working on the tree property. More precisely, assuming the existence of something called weakly compact hypermeasurable cardinal Sy D. Friedman and I proved that in some forcing extension $\aleph_\omega$ is a strong limit cardinal and $\aleph_{\omega+2}$ has the tree property. This improves a result of Matthew Foreman. Now I am trying to make the tree property hold simultaneously at $\aleph_{\omega+2}$ and at many $\aleph_n$'s, for $n < \omega$, from the same assumptions. Getting the tree property at many (more than one) cardinals simultaneously can be very interesting but also very hard. There are still many open question in this field although it is an old subject, for example, as far as I know, it is still open whether the tree property can at the same time hold at $\aleph_{\omega+1}$ and $\aleph_{\omega+2}$, or, is it consistent with ZFC to have the tree property at each $\aleph_n$, $1 < n < \omega$, and $\aleph_{\omega+2}$?

For more information on the tree property and most interesting relevant results I refer you to the following:

[1] U. Abraham, *Aronszajn trees on $\aleph_2$ and $\aleph_3$*, Annals of Pure and Applied Logic (1983), Vol. 24, 213-230..

[2] J. Cummings, M. Foreman *The tree property*, Advances in Mathematics (1998), Vol. 133, 1-32.

[3] M. Magidor, S. Shelah. *The tree property at successors of singular cardinals.* Archive for Mathematical Logic (1996), Vol. 35, 385-404.

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I work in the area of descriptive set theory, which loosely expressed might be defined as the study of concrete and simply definable sets in standard spaces such as $\mathbb{R}$ or ℓ^2. Of course the phrase *simply definable* is a non-technical term which can have various meanings. For a logician with a very constructive bent, *simply definable* might mean something like a predicative definition in the language of set theory without the use of any parameters.

In actual fact, for a descriptive set theorist the reasonably definable sets are things like Borel sets – those appearing in the smallest σ-algebra containing the open sets. More generously, under suitable large cardinal assumptions, one might allow the sets which appear in $L(\mathbb{R})$ – the collection of sets generated by closing the reals under certain primitive operations and transfinite recursion. In either setting, the sets we consider are certainly at least fairly concretely describable from a real number and an ordinal parameter. In any case, the restrictive nature of descriptive set theory is that one would relatively uninterested in say an arbitrary well order of the reals, summoned into existence by the axiom of choice. As a practical matter the restriction to more concrete or more definable sets actually means that there are structural properties one can hope to appeal to, and typically one is working with sets which one might reasonably hope to prove Lebesgue measurable.

In recent years my work has become increasingly involved with the study of equivalence relations. At the most basic structural level problems appear which have no analogue in the classical study of the Borel structure of *subsets* of Polish spaces.

Theorem 4.1 *(Classical) Any standard Borel space has cardinality one of:*

$$0, 1, 2, 3, ..., \aleph_0, 2^{\aleph_0}.$$

Moreover, any two standard Borel spaces of the same cardinality are Borel isomorphic.

The first part of this theorem holds for the cardinality of the number of equivalence classes of a Borel equivalence relation, as was shown in Jack Silver in the late 70's.

Definition 4.2 *An equivalence relation E on a standard Borel space X is said to be Borel if it is in the σ-algebra generated by the Borel rectangles, $A \times B$, where A and B are Borel subsets of X.*

Theorem 4.3 *(Silver) If an equivalence relation E is Borel and it has uncountably many equivalence classes, then it has $2^{\aleph_0}$ many classes.*

However the *moreover* part of the classical theorem fails. Among the Borel equivalence relations with uncountably many classes there is no simple or natural catalogue, and under any reasonable notion of isomorphism there are many, many non-isomorphic Borel equivalence relations with uncountably many classes. It is a major research project in current descriptive set theory to understand the structure of the Borel equivalence relations with uncountably many classes. Typically this is organized around the notion of Borel reducibility:

Definition 4.4 *For E, F Borel equivalence relations on X, Y, we say that E is Borel reducible to F,*

$$E \leq_B F,$$

if there is a Borel function $\theta : X \to Y$ such that for all $x_1, x_2 \in X$

$$x_1 E x_2 \Leftrightarrow \theta(x_1) F \theta(x_2).$$

Just to give a very brief flavor of the area, let $\mathrm{id}(\mathbb{R})$ denote the identity equivalence relation on the reals and let E_0 denote the equivalence relation of eventual agreement on infinite binary sequences. Using techniques from effective descriptive set theory, notably Gandy-Harrington forcing, Leo Harrington, Alexander Kechris, and Alain Louveau showed in the late 1980's that there is a structural theorem relating these two equivalence relations:

Theorem 4.5 *(Harrington-Kechris-Louveau): Let E be a Borel equivalence relation on a standard Borel space. Then exactly one of:*

(I) $E \leq_B id(\mathbb{R})$;

(II) $E_0 \leq_B E$.

These kinds of *dichotomy* theorems have become bench marks in the area. Many of these are surveyed in *Recent developments in the theory of Borel reducibility,* Dedicated to the memory of Jerzy Los, **Fund. Math.** 170 (2001), 21–52, written by Kechris and myself. A rather different, and in some cases more fruitful, approach is given by trying to understand dynamically what properties of continuous group actions are necessary for the orbit equivalence relation to fail to allow certain kinds of complete invariants. For instance, in **Classification and Orbit Equivalence Relations** (G. Hjorth, Mathematical Surveys and Monographs, 75. American Mathematical Society, Providence, RI, 2000) the condition of *turbulence* was indicated to be the critical factor which will prevent classification by countable models considered up to isomorphism.

For a long while the study of countable Borel equivalence relations represented stark challenges, since the techniques used by set theorists were on the whole inadequate in this context. Following work of Adams and Kechris, ideas from superrigidity have been imported into the subject, and just in 2008 we finally managed to show that there exist continuum many $\leq_B$-incomparable treeable countable Borel equivalence relations.

Perhaps the part of this picture which is most mysterious is the dividing line between group actions and non-group actions. For instance:

Problem 4.6 *If E is a Borel equivalence relation, must we have one of:*
(I) $E_1 \leq_B E$ (where E_1 is the equivalence relation of eventual agreement on infinite sequences of reals);
(II) there is a Polish group G acting continuously on a Polish space with induced orbit equivalence relation E_G having $E \leq E_G$?

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It is an old result of Baumgartner that the consistency of a supercompact cardinal implies the consistency of the proper forcing axiom (PFA). For my doctoral thesis, I am trying to prove a theorem of the following form: It is consistent that there exists a model with very large cardinals (in the range of hyperstrongs or beyond), but there is no proper forcing extension of that model in which PFA (or rather: a large fragment of PFA) holds. Theorems like the above support the common belief that the consistency strength of PFA actually equals that of a supercompact cardinal, as any "natural" way to force (large fragments of) PFA would be via a proper iteration of proper forcings. Techniques and topics involved include the following:

- (fragments of) PFA
- Constructibility
- Forcing, Iterated Forcing, Forcing L-like properties
- Large Cardinals and Elementary Embeddings
- Fragments and Generalisations of Condensation

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I am interested in the following topics:

– Large cardinals and the continuum function on regulars, which are supposed to remain large (consistency results).

– Continuum function on singular cardinals of countable/uncountable cofinality (consistency results).

– Definable wellorders on large cardinals.

All these topics can be analysed from the point of the optimality of the assumptions used (e.g. $o(\kappa) = \kappa^{++}$ vs. $P_2(\kappa)$-hypermeasurable), or from the point of general feasibility (controlling continuum function on singulars).

Wellorder is a new topic which combines the properties of $L[E]$-like models and general lifting arguments.

For more please see:

[1] Sy D. Friedman, R. Honzik, *Easton's theorem and large cardinals*, Annals of Pure and Applied Logic **154**,3 (2008) 191–208.

[2] R. Honzik, *Global singularization and the failure of SCH*, Annals of Pure and Applied Logic, to appear (available as an article in print on APAL website).

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My research interest is Descriptive Set Theory especially determinacy, forcing absoluteness and their connections with large cardinals and inner model theory.

Currently I am working on Blackwell determinacy. *Blackwell games* are infinite games with imperfect information while Gale-Stewart games are infinite games with perfect information. Donald Martin [1] proved that the Axiom of Determinacy (AD) implies the Axiom of Blackwell Determinacy (Bl-AD) and conjectured the converse. With David de Kloet and Benedikt Löwe, I [2] introduced the Axiom of Real Blackwell Determinacy (Bl-$\mathrm{AD}_\mathbb{R}$) and proved that Bl-$\mathrm{AD}_\mathbb{R}$ implies the consistency of AD, hence by Gödel's Incompleteness Theorem, the consistency of Bl-$\mathrm{AD}_\mathbb{R}$ is strictly stronger than that of AD. I am now working with Woodin on the problem whether Bl-$\mathrm{AD}_\mathbb{R}$ implies $\mathrm{AD}_\mathbb{R}$ under ZF+DC and we are completing the proof of it. By the result of Solovay [3], $\mathrm{AD}_\mathbb{R}$+DC implies the consistency of $\mathrm{AD}_\mathbb{R}$, hence the assumption for this problem is not optimal and one cannot derive the equiconsistency between ZF+$\mathrm{AD}_\mathbb{R}$ and ZF+Bl-$\mathrm{AD}_\mathbb{R}$ by solving the above problem positively. Woodin conjectured that they are equiconsistent and I am going to work on this conjecture with the technique of core model induction.

[1] D. Martin, *The determinacy of Blackwell games*, J. Symbolic Logic, **63 (4)** (1998) 1565–1581.

[2] D. Ikegami, D. de Kloet, B. Löwe, *The Axiom of Real Blackwell Determinacy*, Submitted to Fundamenta Mathematicae, available at http://www.illc.uva.nl/Publications/ResearchReports/PP-2009-17.text.pdf.

[3] R. Solovay, *The independence of DC from AD*, In Cabal Seminar 76-77 (Proc. Caltech-UCLA Logic Sem., 1967-77), volume 689 of Lecture Notes in Math., Springer, Berlin, (1978) 171-183/

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I am interested in infinite combinatorics. My main project for the past few years has been to construct forcings along morasses. More specifically, I have been interested in constructing systems of complete embeddings between ccc forcings [1]. Then the limit is ccc again. In this context, I got recently interested in the forcing construction of Vera Fischer's PhD thesis [2] which she uses to prove the consistency of $\mathfrak{b} = \kappa < \mathfrak{s} = \mathfrak{a} = \kappa^+$. One step of her iteration can be described as follows [3]: Let κ be a regular cardinal such that $cov(\mathcal{M}) = \kappa$ and $\forall \lambda < \kappa \ (2^\lambda \leq \kappa)$. Then for every unbounded directed family $\mathcal{H}$ of size κ there is an ultrafilter U such that the relatived Mathias forcing $\mathbb{M}(U)$ preserves the unboundedness of $\mathcal{H}$. If one wants to use this forcing to construct a system of complete embeddings, the following question arises: Is there in the generic extension an ultrafilter $\tilde{U}$ with the same properties such that $\mathbb{M}(U)$ can be completely embedded into $\mathbb{M}(\tilde{U})$?

[1] B. Irrgang, *Morasses and finite support iterations*, Proc. Amer. Math. Soc. 137 (2009), 1103-1113.

[2] V. Fischer *The consistency of arbitrarily large spread between the bounding and the splitting numbers* doctoral dissertation, June 2008, York University.

[3] V. Fischer, B. Irrgang *A note on non-dominating ultrafilters* , available on my webpage.

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My work focuses on *indestructibility results* for large cardinals, and on *forcing axioms.* I am particularly interested in the *strongly unfoldable* cardinals, introduced by Villaveces [Vil98] and independently by Miyamoto [Miy98] as the H_{κ^+}*reflecting* cardinals. Strongly unfoldable cardinals are very low in the large cardinal hierarchy, yet over the last few years my work and that of others has shown that they can serve as a highly efficacious substitute for the much larger *supercompact* cardinals:

Indestructibility. In [Joh08], I adapted Laver's landmark indestructibility result for supercompact cardinals and made strongly unfoldable cardinals highly indestructible. Hamkins and I improved this and showed in [HJ10] how to make a strongly unfoldable cardinal κ indestructible by all $<\kappa$-closed κ^+-preserving forcing.

Forcing Axioms. In my dissertation I adapted Baumgartner's relative consistency proof of the Proper Forcing Axiom PFA from a supercompact cardinal and showed that the restricted forcing axiom PFA($\aleph_2$-proper) is consistent relative to the existence of a strongly unfoldable cardinal. Later, Hamkins and I improved this and showed in [HJ09] the same for the conjunction of the principles PFA($\aleph_2$-preserving), PFA($\aleph_3$-preserving), and $\mathrm{PFA}_{\aleph_2}$. Using Miyamoto's result in [Miy98], we established the equiconsistency of these three principles with the existence of a strongly unfoldable cardinal.

Resurrection Axioms. Recently, Hamkins and I introduced a new class of forcing axioms, the Resurrection Axioms. In our forthcoming paper, we illustrate how the Resurrection Axioms are tied to forcing axioms such as MA and BPFA, how they affect the size of the continuum, and we also prove their equiconsistency with the existence of an *uplifting* cardinal, a large cardinal notion much weaker than a Mahlo cardinal.

[HJ09] Joel David Hamkins and Thomas A. Johnstone. The proper and semiproper forcing axioms for forcing notions that preserve $\aleph_2$ or $\aleph_3$. *Proc. Amer. Math. Soc.*, 137(5):1823–1833, 2009.

[HJ10] Joel David Hamkins and Thomas A. Johnstone. Indestructible strong unfoldability. *to appear in the Notre Dame Journal of Formal Logic*, 2010.

[Joh08] Thomas A. Johnstone. Strongly unfoldable cardinals made indestructible. *J. Symbolic Logic*, 73(4):1215–1248, 2008.

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I am currently an assistant professor at the University of Colorado in Boulder. Before coming here I was a postdoc at the University of Wisconsin in Madison, and a PhD student at the University of Michigan in Ann Arbor.

Most of my work in Set Theory has been on certain maximal almost disjoint families of sets. Mostly on confinitary groups. These are subgroups of the symmetric group on the natural numbers, Sym(N).

An element f in Sym(N) is cofinitary if either it has only finitely many fixed points, or if it is the identity.

A subgroup G of Sym(N) is cofinitary if all of its elements are cofinitary.

A subgroup G of Sym(N) is a maximal cofinitary group if it is a cofinitary group and is not properly contained in another cofinitary group.

By observing the equivalence:

$n = g^{-1}f(n)$ iff $f(n) = g(n)$ iff $(n, f(n))$ in f intersected with g

you see that cofinitary groups are indeed almost disjoint families of functions (a family of countable sets is almost disjoint if the intersections of pairwise distinct elements are finite)

Two motivating questions now are

(1) What are the possible complexities of these groups,

(2) What are the algebraic properties of these groups.

If we use the standard action of a subgroup of Sym(N) on N we can study the orbit structure of orbits of maximal cofinitary groups. For this we can show that there are at most finitely many orbits, but any combination of finitely many finite orbits and finitely many (but nonzero) infinite orbits, is possible.

The orbit structure of the diagonal action is not known at this point.

W.r.t. to the possible complexities, it was known by work of Su Gao and Yi Zhang that under V = L there exists a maximal cofinitary group with a coanalytic generating set. We improved this result to show that V = L implies the existence of a maximal cofinitary group that is coanalytic.

It is generally believed that no Borel maximal cofinitary groups exist, but at this point in time it is not yet even been shown that there are no closed maximal cofinitary groups.

W.r.t. the isomorphism types of maximal cofinitary groups, the standard construction provides for freely generated groups. With some modification groups that are not free, but still have a lot of freeness in them can be constructed (getting for instance that there is a maximal cofinitary group into which any countable cofinitary group embeds). With a different method I was able to show that there exists a locally finite maximal cofinitary group (a group is locally finite if any finite subset generates a finite subgroup).

The results mentioned above explicitly can be found in the following two papers:

Isomorphism Types of Maximal Cofinitary Groups, Bulletin of Symbolic Logic, September 2009, Volume 15, pp. 300-319.

The Complexity of Maximal Cofinitary Groups, Proceeding American Mathematical Society, Vol. 137 (2009), no. 1, 307–316.

Since leaving Michigan I have also been working in some other areas of logic. For instance on effective randomness, comparing different notions of effective randomness (specifically: separating injective randomness from Martin-Lof randomness).

Lately I have been working on formalizing set theory (the subject I'll give a talk about). I have finished the formalization of a simple example of a cofinitary group.

An Example of a Cofinitary Group in Isabelle/HOL. In: G. Klein, T. Nipkow, and L. Paulson (ed), The Archive of Formal Proofs, http://afp.sourceforge.net/entries/CofGroups.shtml, August 2009, Formal proof development.

All my papers can be found at:
http://www.bartk.nl/files.php

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Stuart C. King
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I'm a first year PhD student studying Set Theory under the supervision of Professor Philip Welch. Unfortunately Philip has been ill recently but when he returns I will be working on problems in Inner Model Theory. Currently I am reading around the subject and building up my background knowledge of Set Theory and Inner Model Theory.

Although I do work in Set Theory during Philip's absence I've mostly been involved in work in Unprovability with Andrey Bovykin, we have recently been proof reading Harvey Friedmans upcoming book on Boolean Relation Theory and the Incompleteness phenomena found within it, and held a week long conference with Friedman to discuss the book.

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Mikolaj Krupski
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I am interested in general topology, descriptive set theory and measure theory. Currently I investigate properties of countably determined/strongly countably determined measures on compact spaces i.e. measures with the following property:
There exists a countable family $\mathcal{F}$ of Borel/Baire sets such that
$\mu(U) = \sup\{\mu(F) : F \subseteq U, F \in \mathcal{F}\}$, for each open set U (see [3]). These considerations are motivated by the problem posted by D.H.Fremlin: Assuming Martin's axiom, does every Radon measure on the first-countable compact space have to be strongly countably determined?

[1] R. Pol, *Note on the spaces $P(S)$ of regular probability measures whose topology is determined by countable subsets*, Pacific J. Math. 100 (1982) 185-201

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My current research interests are among applications of set theory and logic to various branches of mathematics, in particular to general topology and theory of Banach spaces. Below I describe some of my research themes, with links to articles.

I often use the (already well established) method of elementary substructures. In the paper [7] this is an important tool for constructing projections in non-separable Banach spaces. In fact, an interesting class of Banach spaces is defined by means of countable elementary substructures of $\langle H(\theta), \in \rangle$. The method has also been used several times in the articles [4, 10, 9], in the study of compact spaces "with many retractions".

A significant part of my research is devoted to linear orders and topological spaces they induce. For instance, in [5] I found an example of a linearly ordered compact K for which the Banach space $C(K)$ is a counter-example to the subspace problem for some well known class of spaces "with many projections". The paper [9] contains an internal combinatorial characterization of linear orders that induce the so-called *Valdivia compacta*. Earlier, in [4], I found a universal order preserving pre-image for the class of all Valdivia compact lines.

The preprint [8] deals with the problem of finding universal objects for certain classes. There is a nice theory, due to Roland Fraïssé, of universal homogeneous objects (so-called *Fraïssé limits*) for a given class of first order structures. I present category-theoretic approach which gives much more freedom than the model-theoretic one. In particular, I study structures that are represented as limits of sequences of right-invertible morphisms. As an application, assuming CH, I have obtained the existence of a complementably universal Banach spaces for the class of Banach spaces of density continuum and with the so-called *projectional resolution of the identity*. The same assumption gives the existence of a universal pre-image for the class of Valdivia compacts of weight continuum.

Another topic is topological spaces with continuous structures: linear/partial orders, lattices, groups, trees, etc. When having such a structure in a given space, it is often possible to use some Stone-type duality in order to "move" to a different (topologically simpler) structure and to get useful information about the original space. For example, the articles [2, 1, 3] deal with totally disconnected compact distributive lattices, which are dual to partially ordered sets. Many of the results are proved using this duality. The works [12, 6] contain results on Valdivia compact groups, heavily using the classical Pontryagin duality.

Finally, let me mention the article [11], where we studied an interesting class of compacta, defined by means of upper semicontinuous compact-valued maps from subsets of the irrationals. Some of the results required infinitary combinatorics, absouteness arguments and forcing.

[1] U. Abraham, R. Bonnet, W. Kubiś, *Free Boolean algebras over well quasi-ordered posets*, Algebra Universalis **58** (2008) 263–286

[2] U. Abraham, R. Bonnet, W. Kubiś, M. Rubin, *On poset Boolean algebras*, Order **20** (2003) 265–290

[3] R. Bonnet, L. Faouzi, W. Kubiś, *Free Boolean algebras over unions of two well orderings*, Topology Appl. **156** (2009) 1177–1185

[4] W. Kubiś, *Compact spaces generated by retractions*, Topology Appl. **153** (2006) 3383–3396

[5] W. Kubiś, *Linearly ordered compacta and Banach spaces with a projectional resolution of the identity*, Topology Appl. **154** (2007) 749–757

[6] W. Kubiś, *Valdivia compact Abelian groups*, Rev. R. Acad. Cienc. Exactas Fis. Nat. Ser. A Mat. (RACSAM) **102/2** (2008) 193-197

[7] W. Kubiś, *Banach spaces with projectional skeletons*, J. Math. Anal. Appl. **350** (2009) 758–776

[8] W. Kubiś, *Fraïssé sequences: category-theoretic approach to universal homogeneous structures*, preprint, `http://arxiv.org/abs/0711.1683`

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[11] W. Kubiś, O. Okunev, P. Szeptycki, *On some classes of Lindelof Sigma-spaces*, Topology Appl. **153** (2006) 2574–2590

[12] W. Kubiś, V. Uspenskij, *A compact group which is not Valdivia compact*, Proc. Amer. Math. Soc. **133** (2005) 2483–2487

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The focus of my research is the use of set-theoretic results and methods in the study of infinite groups. Examples of these methods are fine structure theory, forcing and (generalized) descriptive set theory.

Bounding the height of automorphism towers Given a group G_0 with trivial center, we can embed it into its automorphism group $G_1 = Aut(G_0)$ by sending each element g to the corresponding inner automorphism ι_g defined by $\iota_g(h) = ghg^{-1}$. Since G_1 is again a group with trivial center, we can iterate this process and, by taking direct limits at limit ordinals, construct the automorphism tower $\langle G_\alpha | \alpha \in \mathsf{On} \rangle$ of G_0. Simon Thomas showed that for each infinite centerless group G_0 of cardinality κ there exists an ordinal $\alpha < (2^\kappa)^+$ such that the above embedding of G_α into $G_{\alpha+1}$ is an isomorphism. We call the least such α the *height of the automorphism tower of* G_0 and define τ_κ to be the least upper bound for the heights of automorphism towers of centerless groups of cardinality κ. The following so-called *automorphism tower problem* is still unsolved: *Find a model M of ZFC and an infinite cardinal $\kappa \in M$ such that it is possible to compute the exact value of τ_κ in M.*

It is known that $\tau_\kappa < (2^\kappa)^+$ and, for uncountable κ, this is the best cardinal upper bound provable in ZFC. Building on work of Itay Kaplan and Saharon Shelah, I found better upper bounds using the fine structure theory of $L(\mathcal{P}(\kappa))$. An example of such a bound is the least α such that $J_\alpha(\mathcal{P}(\kappa))$ and $J_{\alpha+1}(\mathcal{P}(\kappa))$ model the same Σ_1-statements with parameters from $\mathcal{P}(\kappa) \cup \{\mathcal{P}(\kappa)\}$. Next, I want to analyze the relation between these invariants and 2^κ in different models of set theory.

Changing the height of automorphism towers Although the definition of automorphism towers is purely algebraic, it also has a set-theoretic essence, because there are groups whose automorphism tower depends on the model of set theory in which it is computed. Extending results by Joel Hamkins, Gunter Fuchs and Simon Thomas, Gunter Fuchs and I constructed ZFC-models containing groups whose automorphism tower heights can be changed several times by passing to a forcing extension or an inner model in each step. This shows that, in general, the automorphism tower height is not absolute. I work on proving absoluteness statements for certain classes of groups using descriptive set theory.

Automorphisms of Ultraproducts of Finite Symmetric Groups Given a non-principal ultrafilter $\mathcal{U}$ over ω, we define $S_{\mathcal{U}} = \prod_{\mathcal{U}} Sym(n)$ to be the corresponding ultraproduct of all finite symmetric groups. If the continuum hypothesis holds, then $S_{\mathcal{U}}$ is a saturated structure and $Aut(S_{\mathcal{U}})$ has cardinality $2^{\aleph_1}$. In particular, there is an automorphism that is not inner. It is well-known that if $n \neq 6$, then every automorphism of $Sym(n)$ is inner; and consequently, it appears to be difficult to exhibit an *explicit* example of a non-inner automorphism of $S_{\mathcal{U}}$. Simon Thomas and I showed that there is a good reason behind this difficulty by proving that consistently there exists a non-principal ultrafilter $\mathcal{F}$ over ω such that all automorphisms of $S_{\mathcal{F}}$ are inner. Following this, I want to work on the question whether ZFC proves the existence of an $S_{\mathcal{U}}$ with non-inner automorphisms.

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In my graduation thesis titled *Probabilistic models for Set Theory* and in these first months as a PhD student, I studied a family of models for algebraic set theory. In particular I studied the category of the $\mathcal{H}$-sets, where $\mathcal{H}$ is a complete boolean algebra, in particular it's the Boolean algebra obtained by quotienting the algebra of sets of a probability space by the ideal of null sets. $\mathcal{H}$-sets are couples $(X, \|\circ = \circ\|_X)$ where X is a set coming from an underlying model of set theory and $\|\circ = \circ\|_X$ is function from $X \times X$ to $\mathcal{H}$ opportunely defined that quantifies, with values in $\mathcal{H}$, the degree of equality of two possible elements. In particular is given a Topos structure to this category and is exposed a natural numbers object. In particular $\mathcal{H}$-reals are a $\mathcal{H}$-set whose possible elements are random numbers and the valuation function gives simply the class of the set of equality of two random numbers. This structure for the $\mathcal{H}$-set of reals is linked with the model of reals treated by D.Scott in [4]. In particular, with certain choises of the initial probability space, we can obtain a model of set theory in which Continuum Hypothesis doesn't run.

During the work were pointed out some interesting questions. First of all this model is constructed on a probability space and the $\mathcal{H}$-reals are random variables, so it could be interesting to study the relation beetwen set theoretic operations in the model and probabilistic operations in the underlying space. In this sense we'd like to find characterizations for objects and operations involved in the $\mathcal{H}$-real analysis. In particular the study of $\mathcal{H}$-Lebesgue measure could be interesting in the cases where Continuum Hyphotesis fails. Other reasons to study this family of models are the possibility to give a categorial interpretation of probability and the link with the category of sheaves of a probability space. Finally in these first months I am studying the quotient space $\mathcal{H}$ and the structure of Lewis algebra that could be given to it.

Here are some references:

[1] M.E. Hyland, P. T. Johnstone, A. M. Pitts *Tripos Theory*, Mathematical proceedings of the Cambridge Philosophical Society (1980).

[2] S. MacLane, I. Moerdijk *Sheaves in Geormetry and Logic*, Springer (1992).

[3] G. Rosolini *Sheaves* , Ludwig-Maximilians-Universitat, Munchen (1999).

[4] D. Scott *A proof of the Independence of the Continuum Hypothesis* , Mathematical Systems Theory (1967).

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My research is focused on ideals on ω. It is known that ideals on ω play important role in set theory. For example, we have the follwing theorem.

Theorem [5] Let $\mathcal{I}$ be a Borel ideal on ω. $\mathcal{I}$ doesn't satisfy Fatou's lemma if and only if there exists $X \in \mathcal{I}^+$ such that $\mathcal{S} \leq_K \mathcal{I} \restriction X$.

When we investigate ideals on ω, the study of cardinal invariants of ideals is helpful. In [2], the additive number, the uniformity number, the covering number and the cofinality number of ideal $\mathcal{I}$, donoted by $\mathsf{add}^*(\mathcal{I})$, $\mathsf{non}^*(\mathcal{I})$, $\mathsf{cov}^*(\mathcal{I})$ and $\mathsf{cof}^*(\mathcal{I})$ respectively, are introduced and investigated. In [1, 3], these cardinal invariants and related property are investigated. For example Michael Hrušák, Meza-Alcántara and I prove the following theorem:

Theorem [3] $\mathsf{non}^*(\mathcal{I}) = \omega$ or $\mathsf{non}^*(\mathcal{ED}_{fin}) \leq \mathsf{non}^*(\mathcal{I})$ for every Borel ideal $\mathcal{I}$ on ω.

Theorem [3] If $\mathcal{I}$ is an F_σ-ideal, then $\mathsf{non}^*(\mathcal{I}) \leq \mathfrak{l}$.

Now I'm interested in investigattion of ideals on ω via the cardinal invariants of ideals and forcing notion related with ideals as $\mathbb{M}_{\mathcal{I}^*}$, Mathias forcing with ideals and $\mathbb{L}_{\mathcal{I}^*}$, Laver forcing with ideals.

[1] B. Farkas, L. Soukup, *More on cardinal invariants of analytic P-ideals*, preprint.

[2] F. Hernandez-Hernandez, M. Hrusak *Cardinal invariants of P-ideals*, Canadian Journal of Mathematics, 59, No 3, (2007), 575–595.

[3] Michael Hrušák, David Meza-Alcántara and Hiroaki Minami, *Pair-splitting, pair-reaping and cardinal invariants of F_σ-ideals*, Accepted in the Jorunal of Symbolic Logic.

[4] Michael Hrušák and Hiroaki Minami, *Mathias forcing and Laver forcing associated with ideals*, preprint.

[5] Sławomir Solecki, *Filters and sequences*, Fundamenta Mathematicae, 163, No 3, (2000), 215–228.

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Brice Minaud
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I am a second-year PhD student in Set Theory under the joint direction of Ilijas Farah, at York University (Toronto), and Boban Veličković, at Université of Paris 7. My research interests are focused on trying to find new results in C*-algebras using tools from Set Theory.

The link between Set Theory and C*-algebras is recent. Some important problems in C*-algebras, such as Naimark's problem, have been solved under CH using, for instance, set theoretic results on ultrafilters of $\mathcal{P}(\omega)/FIN$ [2]. Until recently, most results of this kind were limited to using simple set theoretic principles, such as CH or $\diamond$. However it is likely that more results could be obtained with more advanced Set Theory. This approach has already proved fruitful [3]. In [1], Nik Weaver lists a number of C*-algebraic problems that he conjectures can be solved in this way.

I am currently investigating the question of whether pure states on $\mathcal{B}(H)$, the set of bounded operators on a Hilbert space H, are pure iff they are diagonalizable, as stated in [4]. Relevant questions are the relationships between pure states, quantum filters (a filter $\mathcal{F} \subseteq \mathcal{P}(\mathcal{B}(H))$ is said to be a *quantum filter* iff $||p_1p_2\dots p_n|| = 1$ for all $p_1, p_2, \dots p_n \in \mathcal{F}$), and ultrafilters on the naturals (as they appear in the notion of diagonalizable).

[1] Nik Weaver, Set theory and C*-algebras, *Bull. of Symbolic Logic*, vol. 7 (1), 1-20, 2007.

[2] Charles Akemann, Nik Weaver, *Consistency of a counterexample to Naimark's proble*, Proceedings of the National Academy of Science, vol. 101 (20), 7522-7525, 2004.

[3] Ilijas Farah, *All automorphisms of the Calkin algebra are innere*, preprint, arXiv:0705.3085, 2007.

[4] Joel Anderson, *A conjecture concerning the pure states of B(H) and a related theorem*, Topics in modern operator theory, Operator Theory: Adv. Appl., vol. 2, pp. 2743, Timisoara/Herculane, 1980.

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Justin Moore
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My primary research interest is in combinatorial set theory and its applications. Most of my work has concerned the combinatorial properties of $\aleph_1$ and rough classification results concerning structures of power $\aleph_1$. Typical examples of this work the following results.

Theorem (PFA) Every Aronszajn line contains a Countryman line.

Theorem (PFA) If C is a Countryman line and A is an Aronszajn line, then A can be embedded into the direct limit of the alternating product $C \times (-C) \times \ldots \times -C$.

Theorem There is a regular non separable topological space which does not contain an uncountable discrete subspace.

The combinatorics involved in the proofs of the first two results is closely connected to the proof that the bounded form of PFA implies that $2^{\aleph_0} = \aleph_2$. Moreover, it is currently open whether the conclusions of these theorems imply that $2^{\aleph_0} = \aleph_2$. I am also generally interested in understanding the equality $2^{\aleph_0} = \aleph_2$ and when it is a consequence of seemingly unrelated assertions in set theory.

Theorem The formulations of OCA given by Abraham-Rubin-Shelah and by Todorcevic taken in conjunction imply $2^{\aleph_0} = \aleph_2$.

Theorem The bounded form of PFA implies that $2^{\aleph_0} = \aleph_2$ and that there is well ordering of $H(\omega_2)$ which is Δ_2-definable in the structure $(H(\omega_2), \in)$ with parameters.

I am also interested in applications of set theory to other areas of mathematics and I will finish by describing an ongoing project. If we let $\mathscr{T}$ denote the collection of all variable free terms in a language with a binary operation and a constant symbol, then Thompson's group F acts (partially) on $\mathscr{T}$ by re-associating. A long standing open problem is whether this action admits a *finitely additive invariant probability measure* (this is equivalent to whether F is *amenable*). I have proved that, for some constant C, a C^{-n}-Følner set for this action must have at least $\exp_n(0)$ elements (where $\exp_n(0)$ is the *tower function*). I moreover conjecture that the existence of such a mean can be derived from the finite *left self distributive algebras* studied by Laver. These so called *Laver tables* allow one to construct — in the presence of a very strong large cardinal hypothesis — non trivial enumerations of $\mathscr{T}$ which are compatible with the action of F. These enumerations have fast growth properties which we know by the above result are likely to be shared by the Følner function of F (if it is amenable).

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Part of the progress in the study of forcing axioms includes the search for restricted forms of these axioms imposing limitations on the size of the real numbers. For example, it was proved by Justin Moore that BPFA implies $2^{\aleph_0} = \aleph_2$. Currently there are several proofs known of this implication (and, more generally, of the weaker fact that PFA implies $2^{\aleph_0} = \aleph_2$), but all of them involve applying the relevant forcing axiom to a partial order collapsing ω_2. Therefore, it becomes natural to ask whether or not the forcing axiom for the class of all proper cardinal–preserving posets, or even the forcing axiom for the class of all proper posets of size $\aleph_1$ (which we will call PFA(ω_1)), implies $2^{\aleph_0} = \aleph_2$. This is the main question which has inspired my more recent work: In collaboration with D. Asper, I approached this question by building models in which a number of the consequences of BPFA and PFA(ω_1) concerning the behavior of club–sequences on ω_1 hold and at the same time the continuum is arbitrarily large. In fact, we developed a new method for constructing forcing notions and used it to prove that *measuring* as well as a forcing axiom for a certain subclass of small partial orders (i.e, of cardinality ω_1) and which implies $\neg\mho$ are both consistent with $2^{\aleph_0} > \aleph_2$. One related problem that I would like to solve is the consistency of *measuring* together with CH.

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My area of research is the Combinatorics of Singular Cardinals with an emphasis on Cardinal Arithmetic. I am insterested in connections between the PCF conjecture and more standard combinatorics. During this workshop I would like to solve a question which is related to the PCF conjecture. I state it below.

The λ-Erdös cardinals, defined by being the first cardinals κ such that $\kappa \to (\lambda)_2^{<\omega}$, are well known. It not so well known that, in many occasions, they are equivalent to the λ-*Hajnal* cardinals: the first cardinals κ such that $\mathrm{Free}(\kappa, \lambda)$.

The above notation $\mathrm{Free}(\kappa, \lambda)$ means that for every algebra $f : [\kappa]^{<\omega} \to \kappa$ on κ there exists a free set X of order-type λ, and *free* means that for every γ in X, γ is not in the algebraic closure of $X \setminus \{\gamma\}$.

The Erdös and Hajnal cardinals are equivalent in L and $L[U]$. To establish this, ordinal definability plus a technical condensation property are enough [1]. They are not equivalent in general, one can have $\mathrm{Free}(\aleph_\omega, \omega)$ in V. However, this does imply that $\aleph_\omega$ is measurable in an inner model [2], as should be expected since we are saying that $\aleph_\omega$ has cofinal free sets, hence was a Ramsey cardinal and its cofinality was changed to ω.

The question I would like to solve in this workshop is the following:

Question: In $L[U][G]$, where U is a normal ultrafilter and $G = \{\kappa_n\}_{n\in\omega}$ is a Prikry sequence, given an arbitrary *unary* function $f : \kappa \to \kappa$ are there infinite free sets $X = \{\gamma_n\}_{n\in\omega}$ whose elements are of the form $\gamma_n = \sup(N \cap \kappa_n^{+n})$, where N is an internally approachable submodel?

The Fine Structure of $L[U]$ should imply a **negative** answer as it does for elements of the form $\gamma_n = \sup(N \cap \kappa_n)$ and $\gamma_n = \sup(N \cap \kappa_n^+)$ [3].

[1] K. J. Devlin, J. B. Paris, *More on the Free Subset Problem*, Annals Math. Logic 5 (1973), 327-336

[2] P. Koepke, *The consistency strength of the Free-Subset property for ω_ω*, Journal of Symbolic Logic 49 (1984), 1198-1204

[3] L. Pereira, *Combinatoire des Cardinaux Singuliers et Structures PCF*, Ph.D. thesis, September 2007, University of Paris VII- Denis-Diderot, Paris

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My research project is mainly focused on Cardinal Invariant for greater Cardinal.

Most of the Cardinal invariant are define in the real line, some of this definition are easily generalizable; For instance the unbonding number is the minimal cardinal of an ubonding subset of the set of function from omega to omega, so the obvious generalization take larger regular cardinal at the place of omega.

There are some surprising result when we look at the generalized version, as for the spliting number [1] and interesting construction to realize some of the well-know inequality for the cardinal invariant in the real line [2].

I'm interested to continue this study for some other cardinal invariant.

[1] J. Zapletal,*Splitting number at uncountable cardinals*, Journal of Symbolic Logic. (1997) 35–42.

[2] J. Cummings, S. Shelah, *Cardinal invariants above the continuum*, Annals of Pure and Applied Logic **75** (1995) 251–268.

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I am currently looking at combinatorial results on Suslin trees, club guessing and variants of the $\diamondsuit$ principle. Jensen formulated the axiom $\diamondsuit$, a strengthening of CH, to show that ω_1-Suslin trees exist in the constructible universe.

Ostaszewski then formulated the axiom $\clubsuit$, a weakening of $\diamondsuit$ that can hold with or without CH, and the relative consistency of (CH + $\neg\clubsuit$) was established by Jensen, and that of ($\clubsuit$ + $\neg$CH) was established by Shelah.

Later, Kojman and Shelah (and later, others) developed a body of ZFC results on club guessing, a modification of Ostaszewski's $\clubsuit$ principle, where it is restricted to guessing closed unbounded sets.

It is consistent that there is no club guessing sequence on ω_1, but at higher successor cardinals the situation is quite different: club guessing sequences exist in ZFC, $\diamondsuit$ alone does not guarantee the existence of a Suslin tree, and a fairly recent result of Shelah's shows that $2^\lambda = \lambda^+ \rightarrow \diamondsuit_{\lambda^+}$. Motivated by this latter result and its proof, I have been considering new approaches to the existing set of results on club guessing and Suslin trees at the successors of uncountable cardinals. This is a work in progress. Shelah's book Cardinal Arithmetic is a good source of results on such matters. If $S \subseteq \lambda$ is a stationary set of limit ordinals then the relevant definitions are as follows:

($\diamondsuit_\lambda(S)$) there exists a sequence $\langle A_\delta : \delta \in S\rangle$ such that for any $X \subseteq \lambda$ there are stationary many δ with $X \cap \delta = A_\delta$.

($\clubsuit_\lambda(S)$) there exists a sequence $\langle A_\delta : \delta \in S\rangle$ with $A_\delta \subseteq \delta$ unbounded such that for any unbounded $X \subseteq \lambda$ there are stationary many δ with $A_\delta \subseteq X$.

(Club guessing on S) there exists a sequence $\langle A_\delta : \delta \in S\rangle$ with A_δ a closed unbounded subset of δ such that for any closed unbounded $C \subseteq \lambda$ there are stationary many δ with $A_\delta \subseteq C$.

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Karen Räsch
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As a graduate student in mathematics I'm looking forward to seeing into a lot of different topics in set theory.

I take an interest in constructibility theory, forcing and its applications, infinite Ramsey theory and nonstandard analysis. Although not knowing a lot about it (yet), I'm also interested in descriptive set theory and applications of set theory to topology.

My main interest is **Prikry Forcing**. Starting from a measurable cardinal κ, while preserving all cardinals, Prikry-type forcings add an ω-sequence (called Prikry Sequence) that is cofinal in κ. This works out well, because of two orders on $\mathcal{P}$ that interact very nicely such that the second partial ordering compensates the lack of closedness of the partial ordering on $\mathcal{P}$ forming the forcing. I will refer to this interplay as the Prikry Property.
Recently Gitik, Kanovei and Koepke proved that every set in the generic extension by a classical Prikry forcing is constructibly equivalent to a subsequence of the corresponding Prikry sequence. (This result has not been published so far.) In my diploma thesis I investigate whether it is possible to modify the definition of tree Prikry forcing in such a way that still the Prikry property and the connection between the submodels of the generic extension and subsequences of the Prikry sequence hold. Furthermore the modified Tree Prikry Forcing is supposed to permit only the two trivial submodels of the associated generic extension.

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In my upcoming dissertation, I study the effect of square-like principles on diamond and non-saturation.

1. Diamond. A 35 years old theorem of Gregory states that GCH entails $\diamondsuit^*_{E^{\lambda^+}_{<\mathrm{cf}(\lambda)}}$ for every cardinal λ, and a subsequent theorem of Shelah improves this to: GCH entails $\diamondsuit^*_{E^{\lambda^+}_{\neq \mathrm{cf}(\lambda)}}$ for every cardinal λ. (For missing terms, see online version of [10].)

Since then, a lot of work has been done on determining the exact interplay between GCH-type assumptions and the validity of diamond over stationary sets. One of the few questions that remained open was whether GCH is consistent with the failure of $\diamondsuit_S$ for a set $S \subseteq E^{\lambda^+}_{\mathrm{cf}(\lambda)}$ that reflects stationarily often.

It turned out that the answer is negative: Let Refl_λ denote the assertion that every stationary subset of $E^{\lambda^+}_{\mathrm{cf}(\lambda)}$ reflects; then the effect of square-like principles on diamond is summarized in the following.

Fact. For a singular cardinal λ:

(1) by [12], GCH+$\square^*_\lambda \not\Rightarrow \diamondsuit^*_{\lambda^+}$;

(2) by [9], GCH+$\mathrm{Refl}_\lambda + \square^*_\lambda \Rightarrow \diamondsuit^*_{\lambda^+}$;

(3) by [9], GCH+Refl_λ+$\mathrm{SAP}_\lambda \not\Rightarrow \diamondsuit^*_{\lambda^+}$;

(4) by [9], GCH+Refl_λ+$\mathrm{SAP}_\lambda \Rightarrow \diamondsuit_S$ for every stationary $S \subseteq \lambda^+$;

(5) by [1], GCH+Refl_λ+$\mathrm{AP}_\lambda \not\Rightarrow \diamondsuit_S$ for every stationary $S \subseteq \lambda^+$.

Here, AP_λ stands for the *Approachability Property*, whereas, SAP_λ – the *Stationary Approachability Property* – is yet another consequence of $\square^*_\lambda$, isolated in [9].

2. Non-saturation. One of the most famous consequences of diamond is the non-saturation of the nonstationary ideal. By Shelah, $\mathrm{NS}_{\lambda^+} \restriction S$ is non-saturated for every stationary $S \subseteq E^{\lambda^+}_{\neq \mathrm{cf}(\lambda)}$, and so, as in diamond, this line of research focuses on stationary subsets of $E^{\lambda^+}_{\mathrm{cf}(\lambda)}$. By Foreman, building on the work of Woodin, it is consistent (modulo large cardinals) that $\mathrm{NS}_{\aleph_{\omega+1}} \restriction S$ is saturated for a non-reflecting stationary subset $S \subseteq E^{\aleph_{\omega+1}}_\omega$. In my dissertation, a result of complementary nature is established:

Theorem. ([9]) Suppose that λ is a singular cardinal, and that S is a given stationary subset of $E^{\lambda^+}_{\mathrm{cf}(\lambda)}$.

If SAP_λ holds and S reflects stationarily often, then $\mathrm{NS}_{\lambda^+} \restriction S$ is non-saturated. We also dealt with successors of regulars, obtaining:

Theorem. ([11]) Suppose that λ is a regular cardinal, and that S is a given stationary subset of $E^{\lambda^+}_\lambda$.

If $\curlywedge^-(\kappa, S)$ holds for some cardinal $\kappa < \lambda$, then $\mathrm{NS}_{\lambda^+} \restriction S$ is non-saturated.

Here, $\curlywedge^-(\kappa, S)$ is a certain weakening of the generalized club guessing principle introduced by König, Larson and Yoshinobu in [2]. This weakening is a by-product of an answer given in [11] to a question from [2], namely, that GCH is indeed consistent with the failure of such club guessing.

An answer to another question from [2] appears in [1]; a nice by-product of the latter, reads as follows.

Proposition. Suppose $\aleph_0 < \kappa < \lambda$, and S is a given stationary subset of $E^{\lambda^+}_\kappa$; Then TFAE:

(1) $\diamondsuit_S$ holds;

(2) $2^\lambda = \lambda^+$ and there exists a stationary $\mathcal{X} \subseteq [\lambda^+]^\kappa$ such that:

(2.1) $x \mapsto \sup(x)$ is an injection from $\mathcal{X}$ to S;

(2.2) for all $x, y \in \mathcal{X}$, $x \subset y$ iff $\sup(x) \in y$;

(2.3) $\diamondsuit_{\mathcal{X}}$ holds.

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My research area is in Metamathematics of Set Theory, more specifically on Axiomatic Theories of Truth with Set Theory as base theory. My advisor is Alessandro Andretta (University of Turin, Italy).

The investigations on Axiomatic Theories of Truth arose from Tarski's theorem on undefinability of truth [7]. It follows from it and from Beth's definability theorem that no first-order theory can be extended by the definition of a new predicate symbol that represents truth into the language of the theory itself. As a consequence, the axiomatic way to handle truth, extending the theory with a a predicate whose intended meaning is "truth" (the *truth-predicate*), has to provide an axiomatization weaker than a full definition of it (the *truth-theory*).

Since Tarski's work, many alternative axiomatizations of the truth-predicate have been investigated: a common feature is the choice of Peano Arithmetic (or of some similar axiomatization of Number Theory) as *base theory*, i.e. the theory which encodes syntax and which has to be extended with the axioms for truth. Also the semantical counterpart of the truth-predicate, the so-called *satisfaction classes*, have been the subject of many works in the special case of models of Peano Arithmetic. For a survey see Kotlarski [4], Sheard [6], Halbach [3].

A natural question then arises: what it happens if we add the same truth-theory to a different base theory, specifically to an axiomatization of Set Theory?

We can ask more specific questions in connection with Large Cardinals axioms formulated in terms of the existence of an elementary embedding $j : M \prec M$ from an inner model M into itself. Kunen [5] proved (in some class theory like Morse-Kelley) that the existence of such a $j : V \prec V$ is inconsistent with ZFC. Thus, there was room in order to looking for new axioms that could weaken this inconsistent statement.

Among the others proposals, Corazza [1, 2] layed down some axiomatizations for a unary function symbol $\mathbf{j}$ intended to denote a function $j : V \to V$ which shares as many as possible features with an elementary embedding.

My research goal is now to study the interaction between these elementary embedding theories and classical truth-theories for ZFC.

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My research concentrates on descriptive set theory, including the structure of the real line and properties of Borel functions, and on forcing methods with applications to descriptive set theory.

I am especially interested in the theory of "idealized forcing", developed recently by Zapletal. This is a topological and descriptive analysis of a wide class of forcing notions arising as quotient algebras of the Borel sets in a Polish space modulo some σ-ideal. These forcings give generic extensions determined by a single real, called the generic real. Forcing with an idealized forcing has interesting effects at the real line. Zapletal showed for example that if the ideal is generated by closed sets then evaluating ground-model continuous functions at the generic real gives all reals in the extension. This property is called continuous reading of names.

One particular forcing notion, the Steprāns forcing is connected to the Pawlikowski function P, which is a Baire class 1 but not σ-continuous function. It is a "canonical" example of an idealized forcing without the property of continuous reading of names. I established a tree-reprezentation of the Steprāns forcing and using this reprezentation I showed that the function P is the only obstacle to continuous reading of names, i.e. after extending the topology so that P is continuous, continuous reading of names holds. This lead to a question whether any forcing has continuous reading of names in some extended topology. I answered this negatively by contstructing an idealized forcing which does not have continuous reading of names in any extended topology.

As a by-product of my research in the area of idealized forcing, I produced a new and simplified proof of a dichotomy first proved by Solecki for Baire class 1 functions and then generalized by Zapletal to all Borel functions. The dichotomy says that any Borel function is either σ-continuous or else "contains" the Pawlikowski function P.

Recently, I am examining properties of ideals generated by families of closed sets and forcings associated with them. One example of such forcing is connected to piecewise continuity of a Borel function, where we take the σ-ideal generated by closed sets on which a fixed Borel function is continuous. This yields to a forcing notion, which turns out to be equivalent to the Miller forcing. Another good example is the ideal $\mathcal{E}$ generated by closed sets of measure zero. This ideal has been thoroughly examined by Bartoszyński and Shelah but from a slightly different point of view. I was able to establish new results about the ideal $\mathcal{E}$ and its associated forcing, in particuar I found a game representation of the ideal and this led to a fusion in the forcing $\mathbb{P}_{\mathcal{E}}$. That method turned out to generalize to the class of all ideals generated by closed sets, and yields the Axiom A for this class of forcings.

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I'm mainly interested in inner model theory, descriptive set theory, and the theory of forcing axioms. Inner model theory constructs canonical inner models of set theory. These models are usually constructed from a sequence of extenders (and sometimes also from an additional sequence with strong condensation properties). Such models can be analyzed in great detail. They may be used for consistency strength investigations: typically, one tries to construct a "maximal" canonical inner model, called the core model K, and use a given hypothesis to show that K must have large cardinals of a certain kind. But inner models may also be used to translate statements from descriptive set theory into a more "combinatorial" language: for instance the existence of a certain inner model with a Woodin cardinal which "captures" a given set A of reals implies the determinacy of A. Methods from inner model theory are mixed with methods from descriptive set theory to produce the core model induction which is a necessary ingredient in many contemporary optimal consistency strength results. Subtle implications of forcing axioms like PFA or BMM can also be uncovered with the help of inner model theory and/or the core model induction.

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My research area is descriptive set theory. In particular, I try to apply inner model theory to descriptive set theory and definable equivalence relations.

In my dissertation, I characterized the inner models with representatives in all equivalence classes of thin (i.e. without a perfect set of inequivalent reals) equivalence relations of a given projective complexity in the context of large cardinals or determinacy. Such models correctly compute the corresponding projective ordinal. The techniques for proving this rely on $M_n^{\#}$, i.e. iterable models with Woodin cardinals. I further looked at the question when reasonable forcing does not add equivalence classes to thin projective equivalence relations and showed that this is true for absolutely Δ^1_3 equivalence relations if every set has a sharp, in particular δ^1_2 does not change in this situation.

I am further interested in dichotomy theorems for projective equivalence relations. The Harrington-Kechris-Louveau theorem shows that for any Borel equivalence relation E, either $E_0 \leq E$ or E is smooth, i.e. there is a Borel function assigning reals as invariants. The aim is to find an optimal version of the Harrington-Kechris-Louveau theorem for each projective level. An interesting open question is whether every thin Δ^1_2 equivalence relation is either at least as complicated as E_0 or can be reduced to equality on $2^{<\omega_1}$ by a function in $L(\mathbb{R})$ in the context of determinacy. Currently it is known by work of Hjorth and Miller that in the latter case the equivalence relation can be reduced to equality on 2^{ω_1}. Inner model theory could play a role in the solution of this problem.

In a current project I work on extensions of results in classical descriptive set theory such as uniformization, regularity properties, and results on definable equivalence relations to the spaces ${}^{\kappa}\kappa$, where κ is a uncountable cardinal with $\kappa^{<\kappa} = \kappa$.

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David Schrittesser
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I'm interested in many topics of set theory. I have experience with forcing axioms, descriptive set theory, class forcing, iteration theory, some smaller large cardinals and some idea of Jensen coding.

In my thesis we (that is I and my advisor, Sy Friedman) prove that it is consistent that all sets are Lebesgue-measurable but there is a Δ^1_3-set lacking the Baire property. This result builds on [She84] and [She85], which prove, among other results, that it is consistent that all sets are Lebesgue-measurable but there is a set lacking the property of Baire (a nice proof can also be found in [JR93]). This is achieved by mimicking Solovay's proof in that a technique known as amalgamation is used to make sure that the generic extension has lots of automorphisms - this makes sure all sets are Lebesgue-measurable. The forcing is an iteration (since you happen to add reals, and each time you do you have to amalgamate some more and add more automorphisms) collapsing an inaccessible to ω_1; the set without the Baire property simply consists of "every other Cohen real" added in the course of this iteration. We varied this approach by weaving in a set version of Jensen coding, making sure that the counterexample to the Baire property is definable.

For this we had to find a slightly different type of amalgamation that gets along well with closed (or quasi-closed) forcings. I also had to come up with a property that is shared by all the forcings of the iteration, is iterable (with the right support) and preserved by amalgamation. This is called "stratified forcing" (similar to [Fri94]). This notion has the potential - I hope - to be easily adjustable to a variety of settings. Working on these iteration theorems, I became very curious about the idea of generalizing properness to uncountable models (as has been done in [RS], and various other papers)

The result I just described can be seen as a result about two ideals on the set of reals: the meager ideal and the ideal of null sets - in the sense that e.g. being Lebesgue-measurable means being "regular" with respect to the null ideal, or more precisely, being equal to an open set modulo a null set. It would be interesting to prove results as the one mentioned above, but using other ideals than the ones null and meager sets. It would also be interesting to vary the degree of definability: can we make all Σ^1_7 sets Lebesgue-measurable and have a non-measurable set which is Δ^1_8? And at the same time have the Baire property for all Σ^1_4 sets but not for all Δ^1_5 sets? Of course, using large cardinals (Woodin) some results along these lines are known - but Woodin cardinals affect different ideals and regularity properties of sets of reals in a uniform way, whereas I'm interested in forcing so that the the least level at which irregular sets occur is different for different ideals.

Interestingly, in my thesis we collapse a Mahlo to ω_1; while [She85] shows that at least an inaccessible is needed, it is entirely unclear if we need to assume the existence of a Mahlo.

In my master thesis, I proved that a certain type of forcing axiom is equiconsistent with what I called a light-face indescribable cardinal (the terminology may have changed), see [Sch04]. I'm still interested in forcing axioms and would like to pick up this line of research again.

I'm very curious about applications and therefore would like to learn more about C^*-algebras. I would also like to learn about inner model theory.

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I'm a student of fifth year of Double Degree Program in Computer Science and Mathematics. My fields of interest include theory of computation, general topology, descriptive set theory, infinite combinatorics and mathematical logic. I'm mostly interested in connections between these themes, for example in:

- topological results in theory of computation, e.g. descriptive complexity of languages defined by several classical computation models,
- strong connections between logics and models of computation, e.g. equivalence between MSO-logic and finite automata over various structures, see [2],
- infinite Boolean circuits, understood either as representation of Borel complexity of some set $A \subseteq 2^\omega$ or infinite analogy of finite circuits,
- combinatorial methods in analysis of topology of the Cantor and the Baire spaces.

Problems presented above have arisen at several seminaries and courses at my faculty. In some of them I have found some simple, but potentially interesting results:

First of them is extension of results by Michael Sipser in [1]. He showed some property of all infinite Boolean circuits of finite depth. The finite analog of this property is well known as a Furst, Saxe, Sipser theorem about separation of AC^0

and NC^1. Me and Adam Radziwoczyk-Syta (student of Warsaw University) have extended Sipser's result into all countable Boolean circuits, even those of infinite depth. There is an analogy with the Galvin-Prikry theorem.

Second of them is an analysis of classes $BC(\Sigma_1^0), \Delta_2^0, BC(\Sigma_2^0)$ and Δ_3^0 on 2^ω. I have found some simple property of mappings from full infinite binary tree to $\mathbb{N}$. Each mapping satisfying this property induces some set in Δ_3^0. Moreover, there are four natural subfamilies of such mappings, inducing exactly all sets in $BC(\Sigma_1^0), \Delta_2^0, BC(\Sigma_2^0)$ and Δ_3^0 respectively. The result is strongly related to the theory of ω-regular languages.

My current research is focused on studying descriptive set theory and theory of finite automata from topological and logical point of view.

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I am a first year MSc student at the Faculty of Mathematics in our university. My main interests are set theory, independence results and general topology. I wrote my BSc thesis in the latter topic, about topologies originated in the idea of separate continuity. I have not been in any serious research projects, now I am focusing on learning the important methods of these topics and looking for opportunities to join in researches.

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I am a 5th year master degree student at the University of Wroclaw. I am interested in descriptive set theory, forcing and general topology. Currently I am working on universally Baire operations (see [1]).
Having a set X and $P \subseteq \mathcal{P}(X)$ we define Marczewski measurable sets as follows:

$$A \in Mar(P) \iff \forall p \in P\ \exists q \leq p\ (q \in A \vee q \in A^c)$$

I am investigating properties of P which guarantee that $Mar(P)$ is closed under universally Baire operations. It is known that topological forcing notions P have this property ([4]). In this case $Mar(P)$ are sets with Baire property for some topology τ on X, and universally Baire operations preserves Baire property in all topological spaces. The last fact generalizes Vaught-Schilling theorem, which states that absolutely Δ^1_2 operations preserve the Baire property (see [3] and [4]).

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I am a third-year PhD sudent in Set Theory at the university Paris 7. I am interested in forcing, mainly in the countable iteration of proper forcings.

Preservation theorems are a central tool in forcing theory. A lot of questions about preserving a property by iterated forcing arise naturely. One of the finest forcing property for not collapsing ω_1 was discovered by Saharon Shelah, and is called proper. Proper forcings are very usefull because countable support iteration of proper forcings is proper. Thus we can show the consistency of the Proper Forcing Axiom, PFA, with the assumption that $Cons(ZFC+ \exists\ \kappa$ *supercompact*) It is well know that PFA implies the failure of CH. This result leads to the following informal question : is there a kind of forcing axiom below PFA which is maximal in some sense, and which is consistent with CH?

One natural way to show the consistancy with CH of some fruitfull statement (by example some consequence of PFA) is to find an iterable forcing property such that ω_1 is not collapsed and no new real has been added. We already know that an iteration of proper forcing such that each iterand doesn't add a new real can add a new real. But the new real has some mild property: actually there is a real in the ground model which bounds the new real.

In order to avoid this case, Shelah has strengthened the property of not adding reals by a new condition called Dee complete. But he has to suppose that the forcing are more than proper too: each iterand must be $< \omega_1$- proper. This technique can be used by example to prove the consistency of PID with CH.

This technique has been improved by Todd Eisworth, showing that $< \omega_1$-proper can be weakened. So it seems that there is no optimal technique for iterating forcing without adding reals.

One direction to settle this problem is to find two Π_2 statements which follow from PFA, such that they are both consistent with CH but their conjections implies the failure of CH, thus giving a negative answer to the existence of a kind of maximal forcing axiom consistent with CH. Recently it seems that David Aspero, Paul Larson and Justin Moore have settled this question.

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My research concentrates on interactions between set theory (also descriptive set theory) and analysis; I am fond of set-theoretical tools inspired by analysis (e.g. Todorčevićs oscillation maps) and applications of combinatorial set theory to the theory of Banach spaces especially.

The main object of my investigations is the classical quotient Banach space ℓ_∞/c_0 isomorphically isometric to the space of continuous functions on $\omega^\star$, a Stone space of Boolean algebra $\mathcal{P}(\omega)/\text{fin}$. The structure of $\mathcal{P}(\omega)/\text{fin}$ under PFA was already studied and since the theory of ℓ_∞/c_0 is much bigger than the theory of the Boolean algebra above it seems natural to examine fundamental properties of ℓ_∞/c_0 under PFA and its weaker versions. My current objectives regarding this issue are:
- to find class of nonseparable Banach spaces that can be embedded in ℓ_∞/c_0,
- to build some special operators (e.g. Dugundji operator) on it,

- to show a non-primariness of ℓ_∞/c_0.
It should be mentioned that it is already known that ℓ_∞/c_0 is universal for the class of separable Banach spaces and it is primary when CH is assumed. I hope my efforts combined with approaches from [1] and [3] will bring some new constructions of nonclassical Banach spaces and maybe new combinatorial tools to deal with problems around PFA.

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I am a master's student at Manchester. My current interest lies in the consistency of introducing various large cardinals into ZF and their effect on the set theoretic universe.

My interest above arise from the following consideration: If we try to iterate the successor operation on $\varnothing$, we will go on "indefinitely". We declare a ceiling to "everything " below this ceiling and call this ceiling ω. (Hence, heuristically, we have $V_\omega \vDash$ ZF-Infinity.) If we continue with this iteration with ω, and declaring ceilings and perform the operation successively, the "ultimate" ceiling to these operations performed in succession will be somewhat the limit that ZF can give us in terms of sets. This ceiling is the first inaccessible cardinal κ_0 and as in the case of ω, V_{κ_0} is heuristically a model of ZF. Also heuristically, V_{κ_0} cannot prove that there exists an inaccessible cardinal.

If we perform the operations again, now starting from κ_0, then we can arrive at a second inaccessible κ_1 and have another model of ZF for which an inaccessible cardinal exists. So, the question arises to me as to how rich V_{κ_1} is. What basic questions about cardinal comparisons and arithmetic can be answered in this model?

There has been a wealth of results on the subject of large cardinals. At this moment, I am not in a position to mention which particular aspect of the theory I am likely to work on as I still have a shortage of knowledge on the subject, but my current thoughts seem to bring me into investigating large cardinals in inner models and the existence and forms-which set or ordinal-of large cardinals in countable models of ZF.

My other interests include pure logic and stability theory with some minor interests in applied model theory and recursion theory.

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My research centres around the classification of relational structures (e.g. orders, graphs) and those with extra topological structure (e.g. ordered spaces, Boolean algebras) via the embeddability relation. An embedding is generally an injective structure-preserving map between structures. For example, for linear orders, the ordering is preserved in embeddings. The embeddability relation, $A \leq B$ iff A can be embedded into B, is a quasi-ordering of the structures. I study two aspects of this relation: universal structures, those on the top of the embeddability quasi-order, and internal properties of the embeddability structure.

Universal models are structures which embed all other structures (of the same theory) of the same size. For first-order definable theories, it is known via model theoretic arguments that universal models exist in all cardinals above the size of the language. Considering theories which are not first-order definable and even first-order theories in the absence of GCH usually requires set-theoretic techniques.

When GCH fails, it is often independent whether structures may have universal models in uncountable cardinals. In fact, the techniques used to prove universality results in these cases include forcing and infinite combinatorics. For instance, it is an open question whether there is a model of not CH in which there is a universal triangle-free graph at $\aleph_1$. However, I have recently constructed a model in which CH fails and $2^{\aleph_1}$ is large in which there is a small universal family for triangle-free graphs, namely a family of size $\aleph_2$ which embeds all other triangle-free graphs.

When GCH holds, I have considered structures like posets or trees which omit large chains and graphs which omit large cliques. When "large" is some infinite cardinal, then the theory of these structures is not first-order definable. As an example of such a result, I have proved that there is a universal model for trees of size λ which omit κ-branches if and only if λ is a (strong) limit cardinal with $\lambda > \kappa \geq \aleph_0$ and $\mathrm{cf}(\kappa) > \mathrm{cf}(\lambda)$. Also, I am studying structures like linearly ordered spaces (whose theory is also not first-order definable) under continuous order-preserving embeddings.

I also consider internal properties that the embedding quasi-order may have. For instance, Laver proved that countable linear orders form a well-quasi-order under the embeddability relation. A well-quasi-order is a quasi-order which has no infinite antichains and no infinite descending sequences. Laver's result is proved using Hausdorff's constructive hierarchy of scattered orders (those that do not embed the rationals). We would like to extend such a result to uncountable linear orders, but the notions of "scattered" and "dense" diverge in this case. Namely for uncountable κ, we may consider κ-dense linear orders to be those in which between every two points, there are κ-many points in between and κ-saturated to be those in which between every two sets of size $< \kappa$, there is a point in between. So far, we have a Hausdorff-like constructive hierarchy only for those linear orders which do not embed a κ-dense order.

I am also interested in the antichain structure of embedding quasi-orders. In recent work with P. Schlicht, we have constructed an antichain of κ-trees of size κ for $\mathrm{cf}(\kappa) = \omega$ in the embedding quasi-order where the embeddings only preserve strict order (not necessarily injective).

For a list of my papers, see http://www.logic.univie.ac.at/∼thompson

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My primary field of research is descriptive set theory. In particular, I am interested in Borel and analytic equivalence relations, in Polish groups and their actions, in measure preserving group actions on probability spaces, and in the study of Borel reducibility and orbit equivalence.

The study of equivalence relations has been the center of intense interest within the field of descriptive set theory in the past 20 years, and continues to be a particularly fertile area of research. There are strong connections to other areas of mathematics, in particular to ergodic theory and to operator algebras. My own work has to a large extend focused on analysing naturally occuring isomorphism relations in ergodic theory and in operator algebras from the point of view of descriptive set theory. One such is the notion of *orbit equivalence*, an equivalence relation which plays a central role in ergodic theory. Two measure preserving actions σ_1 and σ_2 of countable groups G_1 and G_2, respectively, acting on a standard Borel probability space (X, μ) are said to be *orbit equivalent* if there is a measure preserving automorphism of (X, μ) that maps σ_1-orbits onto σ_2-orbits almost everywhere. In other words, the two induced orbit equivalence relations are isomorphic (on a set of full measure). In my thesis I showed that not only does the free group F_n on $n > 1$ generators have uncountably many non- orbit equivalent actions, there are in fact so many non-orbit equivalent actions of this group that they can't be classified in a reasonable way using *countable structures*, such as countable groups, graphs, or linear orders, as complete invariants. This work has recently been generalized by Inessa Epstein in her thesis to show that the result holds for any non-amenable countable group.

In a similar way, I have also explored the classification problem for von Neumann factors. Von Neumann factors are the building blocks of the theory of von Neumann algebras, a main branch of the field of operator algebras, and the classification of von Neumann factors has been regarded as one of the most important problems in the field. Together with Roman Sasyk (Buenos Aires), we showed that separable von Neumann factors cannot be classified up to isomorphism by a reasonable assignment of countable structures as invariants. This in effect explains why the problem of classifying factors has turned out to be so difficult, and why the invariants that have been used succesfully in some cases (e.g. injective factors) are necessarily so complicated.

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Todorcevic showed that the reflection of stationary subsets of $[\omega_2]^\omega$ implies that the cardinality of the continuum is bounded by $\aleph_2$ ([4]), or written in modern notation, that

$$\mathrm{WRP}(\omega_2) \longrightarrow 2^{\aleph_0} \leq \aleph_2.$$

Following some work of Velickovic and of Foreman-Todorcevic that generalizes this to higher cardinals, Shelah has recently shown that reflection of stationary subsets of arbitrary $[\kappa]^\omega$ implies that $\theta^{\aleph_0} = \theta$ for all regular cardinals $\theta \geq \omega_2$, or in short that

$$\mathrm{WRP} \longrightarrow (\forall \theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \theta^{\aleph_0} = \theta.$$

So, in particular, WRP implies the Singular Cardinal Hypothesis ([2]).

It has been noted by Woodin in his well known monograph [5] that WRP is not sufficient for giving us the stronger conclusion $\lambda^{\aleph_1} = \lambda$ even for $\lambda = \aleph_2$. We go further: We show that the same weak reflection principle for stationary sets, WRP, used by Shelah getting $\theta^{\aleph_0} = \theta$, will give us the stronger cardinal arithmetic $\theta^{\aleph_1} = \theta$ for all regular $\theta \geq \omega_2$ as long as we add to it the assumption that the ideal NS_{ω_1} of non-stationary subsets of ω_1 is saturated. In short, we show that

$$\mathrm{WRP} + \mathrm{sat}(\mathrm{NS}_{\omega_1}) = \aleph_2 \longrightarrow (\forall \theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \theta^{\aleph_1} = \theta.$$

In fact, we shall show that this assumption will give us a bit strong result,

$$\mathrm{WRP} + \mathrm{sat}(\mathrm{NS}_{\omega_1}) = \aleph_2 \longrightarrow (\forall \theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \diamondsuit_{[\theta]^{\omega_1}}.$$

Another theme is the compactness of infinitary logics introduced by Erdös and Tarski in 1943 ([1]) motivated of course by Gödel's compactness theorem for first order logic. We shall in fact consider only a very weak form of the compactness through a combinatorial principle called Rado's Conjecture.

In 1991, Todorcevic showed that

$$\mathrm{RC} \longrightarrow (\forall \theta = \mathrm{cf}(\theta) \geq \aleph_2)\ \theta^{\aleph_0} = \theta$$

(see [3]). So, in particular RC implies the Singular Cardinals Hypothesis. This RC has the same influence on the cardinal arithmetic as the Weak Reflection Principles, WRP, considered above. We have seen above that supplementing WRP with the assumption that NS_{ω_1} is a saturated ideal gives us stronger consequences on cardinal arithmetic. Thus, it is natural to examine whether RC supplemented by the saturation of NS_{ω_1} would also give us the stronger consequences for cardinal arithmetic similarly with the situation of the reflection principle WRP discussed above.

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[4] S. Todorcevic, *Reflecting stationary sets*, handwritten notes, October 1988.

[5] W. H. Woodin. *The axiom of determinacy, forcing axioms, and the nonstationary ideal*, volume 1 of the *Gruyter Series in Logic and its Applications.* Walter de Gruyter & Co., Berlin, 1999.

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I am mainly interested in the class of $C^{(n)}$-cardinals. These cardinals arise naturally, after the definition of the closed unbounded proper class of ordinals that are Σ_n-correct in the universe i.e. for $n \in \omega$, we let $C^{(n)} = \{\alpha : V_\alpha \preccurlyeq_{\Sigma_n} V\}$. In connection with the standard viewpoint in the study of large cardinal axioms, we consider $j : V \longrightarrow M$ a non-trivial elementary embedding of the universe into some transitive model (with critical point κ) and we are interested in strong closure properties of the model M; in particular, the behavior of the image $j(\kappa)$ of the critical point. In the light of the $C^{(n)}$-classes, it is natural to ask whether this image belongs to any of them i.e. when do we have that $j(\kappa) \in C^{(n)}$?

This question gives rise to a wide class of definitions, the $C^{(n)}$-cardinals, where several large cardinal notions get prefixed by "$C^{(n)}$", with the intended meaning being that, in addition to the standard definition for the cardinal κ, we require that its image under the embedding belongs to the club class $C^{(n)}$. Consequently, we get cardinal notions such as $C^{(n)}$-measurables, $C^{(n)}$-(super)strongs, $C^{(n)}$-supercompacts etc. Although there are several results concerning these cardinals, certain questions still remain unsolved, even at the lowest levels of the $C^{(n)}$-hierarchies e.g. is the least supercompact a $C^{(1)}$-supercompact?

Finally, I am also interested in some related philosophical issues. The problem of justifying the "acceptance" of strong axioms of infinity is a central theme that was already addressed at by Gödel. A dominating idea is to employ reflection principles in order to provide intrinsic justifications of such complicated and strong hypotheses. This idea has been studied to some extent during the last decade and there are several results on the negative side, showing certain limitations of reflection principles (cf. [2], [4]). On the other hand, recent (unpublished) results on $C^{(n)}$-cardinals in connection to Vopěnka's Principle indicate that there might be a bright side of reflection after all (cf. [1]).

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My interests in set theory are both mathematical and philosophical. I'm attending my first year of PhD and since now I am focusing on the study of some consequences of the Forcing Axioms. In particular, in this period I am studying problems related to Shelah's Conjecture (: every Aronszajn line contains a Countryman suborder) and the equivalent Five Element Basis Conjecture (: the orders $X, \omega_1, \omega_1^*, C$ and C^* form a five element basis for the uncountable linear orders any time X is a set of reals of cardinality $\aleph_1$ and C is a Countryman suborder). Moore showed that the Conjectures follow from PFA, but soon after has been discovered by König, Larson, Moore and Veličković that the consistency strength of the hypothesis can be reduced to that of a Mahlo cardinal, instead of that of PFA, whose upper bound is a supercompact cardinal and whose lower bound is a class of Woodin cardinals.

There are many problems related to this subject, that would be worth studying. First of all, a question that arise naturally is: do we really need some large cardinal strength for Shelah's Conjecture? Moreover it is interesting to see which are the influences of this Conjecture on the vardinality of the Continuum, because in the models of PFA, $2^{\aleph_0} = \aleph_2$, but if we do not need the consistency strength of PFA, can we find a model where Shelah's Conjectures holds, but the cardinality of the Continuum is difffers from $\aleph_2$?

Another interesting aspect of Shelah's Conjecture is that, modulo BPFA, it is equivalent to the Coloring Axiom for Aronszajn Trees (: for any partition $T = K_0 \cup K_1$ of an Aronszajn tree T, there is an uncountable set $X \subseteq T$ and $i < 2$ such that $x \wedge y \in K_i$, for all $x, y \in X, x \neq y$; i.e. there is an uncountable subset $X \subseteq T$ such that $\wedge(X)$ is totally contained or disjoint from a fixed color) and to the follwoing assertion on the whole class of the Aronszajn tree ($\mathcal{A}$): if T is a Lipschitz tree, then T^+ in an immediate successor of T in $(\mathcal{A}, \leq)$. Hence the study of Shelah's Conjecture, modulo Forcing Axioms, is usefull in a structural analysis of the class of the Aronszajn trees.

This fild of research should be set in the general frame of the search for the exactly consistency strength of the Forcing Axioms, since a carefully understanding of the strength of the hypothesis nedeed for each theorem would help us to have a clearer picture of the universe of set theory.

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[Todorčević 2001] Stevo Todorčević : *Lipszhitz maps on trees*, Report 2000/01 number 13, Istitut Mittag-Leffler.

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The well known *bounding number* defined as follows:

$$\mathfrak{b} = \min\{|\mathcal{A}| : \mathcal{A} \subseteq {}^{\omega}\omega \ \& \ (\forall f \in {}^{\omega}\omega)(\exists g \in \mathcal{A})(g \not\leq^* f)\}$$

is the minimal cardinality of an unbounded family in the order $({}^{\omega}\omega, \leq^*)$. Also well known is the cardinal invariant $\mathrm{non}(\mathcal{M})$, the minimal cardinality of a non-meager subset of $\mathbb{R}$ (or equivalently of 2^{ω}) and the *splitting number*:

$$\mathfrak{s} = \min\{|\mathcal{A}| : \mathcal{A} \subseteq [\omega]^{\omega} \ \& \ (\forall X \in [\omega]^{\omega})(\exists A \in \mathcal{A})(|A \cap X| = |X \setminus A| = \omega)\},$$

the minimal cardinality of a splitting family. Given two (partial functions) h, g we let $hit(h, g) = \{n \in \omega : n \in \mathrm{dom}(h) \cap \mathrm{dom}(g) \ \& \ h(n) = g(n)\}$. A slightly more obscure cardinal invariant is concerned with hitting partial bounded functions. The formal definition is the following: $\mathfrak{k}$ is the minimal size of a family of functions $\mathcal{A}$ such that each partial function h bounded by the identity, i.e. $(\forall n \in \mathrm{dom}(g))(g(n) \leq n)$, is hit infinitely often by some $f \in \mathcal{A}$, i.e. $|hit(f, h)| = \omega$.

The following holds:

$$\mathrm{non}(\mathcal{M}) = \max\{\mathfrak{k}, \mathfrak{b}\}$$

Since $\mathfrak{s} \leq \mathcal{M}$ this might indicate that it should be provable, that either $\mathfrak{s} \leq \mathfrak{b}$ or $\mathfrak{s} \leq \mathfrak{k}$.

However S. Shelah (see [1]) has proved the consistency of $\mathfrak{s} > \mathfrak{b}$ and M. Hrušák conjectured, that it might be feasible to also prove the consistency of $\mathfrak{s} > \mathfrak{k}$. The idea is to start with a model of CH and use countable support iteration of Matthias forcing $M(\mathcal{U})$ to ω_2. This will clearly make $\mathfrak{s} = \omega_2$ and, if the ultrafilters are chosen carefully, might preserve $\mathfrak{k} = \omega_1$. Also it seems plausible that a similar construction might prove the consisency of $\mathfrak{s} > \mathfrak{b}$ simplifying Shelah's proof somewhat.

So far we have been able to prove that Matthias forcing does not add a bounded (total) function which would be eventually different from each ground model function. If this result could be extended to partial functions and the ultrafilter is chosen carefully to preserve maximality with respect to almost disjointness of certain families, then the cardinal $\mathfrak{k}$ will be preserved in successor steps of the iteration.

[1] Shelah, S. *On Cardinal Invariants of the Continuum*, Axiomatic Set Theory (Baumgartner, Martin, Shelah, eds.). Contemporary mathematics, vol. 31, AMS 1984, pp. 183–207.

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Recently I got interested in Chrisoph Weiss work which he is collecting in his Ph.D. thesis. He was interested in generalization of the tree-property and of ineffability that would fit in a wider context. Magidor and Jech already defined the correct generalization of these notions in the combinatorics over $[\lambda]^{<\kappa}$ with $\lambda \geq \kappa$ and κ inaccessible.

Christoph Weiss has correctly extended the notion of ineffability to the combinatorics over $[\lambda]^{<\kappa}$ where κ is an arbitrary regular cardinal.

In particular these notion are reelvant for my current research:

Definition 4.7 *$D = \{d_X : X \in [\lambda]^{<\kappa}\}$ is a list if $d_X \subseteq X$ for all X.*

$d \subseteq \lambda$ is an ineffable branch for the list if $d \cap X = d_X$ for all $x \in S$ for some S stationary subset of $[\lambda]^{<\kappa}$

Given a list $D = \{d_X : X \in [\lambda]^{<\kappa}\}$ and $\theta \geq \kappa$:

- *M is a slender point for D, if $M \prec H(\theta)$ has size less than κ and if $X = M \cap \lambda$ then $D_X \cap Z \in M$ for all $Z \in M$ of size less than κ.*
- *D is a slender list if for some $\theta > \lambda$ there is a club of slender points for D in $[H(\theta)]^{<\kappa}$.*

It is a simple warm-up exercise to check that if a list has an ineffable branch S, then the set of its slender points includes S modulo a club.

Weiss introduced the following remarkable converse:

Definition 4.8 *$ISP(\kappa, \lambda)$ holds if any slender list on $[\lambda]^{<\kappa}$ has an ineffable branch.*

Moreover if κ is inaccessible it is easily checked that every list on $[\lambda]^{<\kappa}$ is slender. While if κ is not inaccessible it is easy to find a list which has no ineffable branch. However this list may not be slender.

Magidor proved the following:

Theorem 4.9 *κ is supercompact iff for all $\lambda \geq \kappa$, every list D on $[\lambda]^{<\kappa}$ has an ineffable branch.*

Thus Weiss' principle $ISP(\kappa, \lambda)$ seems to capure the extent of supercompactness an arbitrary regular cardinal κ can have. He also had shown the following:

Theorem 4.10 (Weiss, 2009) *Assume κ is supercompact and GCH. Then for any regular $\tau < \kappa$ there is a forcing P such that in the generic extension by P, $ISP(\tau^+, \lambda)$ holds for all $\lambda \geq \kappa$.*

Thus he asked me whether $ISP(\kappa, \lambda)$ implies SCH which in view of the above observations seems rather natural.

Thanks to this suggestion I could prove jointly with him the following remarkable theorem:

Theorem 4.11 (Weiss, Viale, 2009) *PFA implies SCH.*

The theorem is remarkable for two reasons:

- It seems that is the unique theorem I can prove
- The proof splits in two parts:

Theorem 4.12 (Weiss, Viale, 2009) *PFA implies $ISP(\aleph_2, \lambda)$ for all $\lambda \geq \kappa$.*

Theorem 4.13 (Viale, 2009) *$ISP(\aleph_2, \lambda) + MA(\aleph_1)$ imples SCH.*

It should be pointed out that my contribution in the proof of 4.12 is marginal since Weiss already had proved that PFA implies a slight weakening of $ISP(\aleph_2, \lambda)$ (which he calls $ITP(\aleph_2, \lambda)$) and that the techniques used in the proof of the latter theorem are just a refinement of those already isolated by Weiss.

I consider theorem 4.12 remarkable for it opens the way to a new pattern towards a proof that PFA should really be equiconsistent with a supercompact cardinal. In particular we expect that the following should be the case:

Conjecture 4.14 *Assume MM, assume W is a "nice" inner model such that $\kappa = \aleph_2$ is inaccessible in W. Then κ is supercompact in W.*

"nice" is not yet clearly stated, for example it could be W is such that the universe is a forcing extension of W by a "nice"-forcing P (for example P is one of the standard iteration of length κ to prove MM).

In this direction we have ideas and start possibly to have promising results but not yet a definite theorem.

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I am a newly arrived PhD student in Paris working under the supervision of prof Stevo Todorcevic on Ramsey theory and its applications. I am mostly interested in applications of Halpern Lauchli theorem in areas like Ramsey spaces of strong subtrees or in continuous colorings of $\mathbb{Q}^{|k|}$. Also I am looking in the applications of Ramsey theory in the partition calculus of infinite products of finite sets.

[1] S. Todorcevic, *Introduction to Ramsey spaces*, Annals of Mathematical studies

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Currently I am a PhD student of Stefan Geschke in Bonn. I work on a project concerned with cardinal charactaristics that are derived from continuous Ramsey theory. A theorem of Andreas Blass states that if one colours the n-tupels of elements of the Cantor space continuously with m colours, m and n both being finite, then there exists a perfect weakly homogeneous set. "Weakly homogeneous" here means that the colour of an n-tupel only depends on the order of the levels where the branches separate. As an example a tripel can be such that the two leftmost branches separate before the two rightmost do or vice versa. In general an n-tupel has one of $(n-1)!$ possible splitting types.

Now these weakly homogeneous sets generate a σ-ideal and one can ask for its covering number, i.e. the minimal size of a family of weakly homogeneous sets covering the whole space. This is a cardinal characteristic and indeed one that tends to be large. There are two respects in which it is large, its cardinal sucessor has size at least continuum and it is always at least as large as the cofinality of the null ideal and hence at least as large as any cardinal characteristic from Cichoń's diagram. Both facts were found by Stefan Geschke.

Currently it is known that the characteristics for pairs and tripels are small, i.e. $\aleph_1$ in the Sacks model and it is known how to separate them from one another. These are results of Stefanie Frick. Much more is unknown however.

It is unknown although conjectured that generally the characteristic for n-tupels is small in the Sacks model.
It is unknown how the characteristics relate to characteristics in van Douwen's diagram which do not lie below $\mathfrak{d}$, that is $\mathfrak{a}, \mathfrak{i}, \mathfrak{r}$ and $\mathfrak{u}$.

It is unkown whether such a characteristic can be smaller than the continuum when the latter is say $\aleph_3$.

I all too often get intrigued by other kinds of set theory. I however have a certain tendency to prefer combinatorics over logic if this makes sense.

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My Ph.D. thesis dealt with a combinatorial principle that captures supercompactness for small cardinals like ω_2.

For κ regular uncountable and $\lambda \geq \kappa$, let us call $\langle d_a \mid a \in \mathcal{P}_\kappa\lambda\rangle$ a $\mathcal{P}_\kappa\lambda$-*list* if $d_a \subset a$ for all $a \in \mathcal{P}_\kappa\lambda$. A $\mathcal{P}_\kappa\lambda$-list $\langle d_a \mid a \in \mathcal{P}_\kappa\lambda\rangle$ is called *thin* if there is a club $C \subset \mathcal{P}_\kappa\lambda$ such that $|\{d_a \cap c \mid c \subset a \in \mathcal{P}_\kappa\lambda\}| < \kappa$ for all $c \in C$.

By (κ,λ)-ITP we denote the following principle: If $\langle d_a \mid a \in \mathcal{P}_\kappa\lambda\rangle$ is a thin $\mathcal{P}_\kappa\lambda$-list, then there are a stationary $S \subset \mathcal{P}_\kappa\lambda$ and $d \subset \lambda$ such that $d_a = d \cap a$ for all $a \in S$.

As every $\mathcal{P}_\kappa\lambda$-list is thin if κ is inaccessible, we have the following theorem due to Magidor: A cardinal κ is supercompact iff κ is inaccessible and (κ,λ)-ITP holds for all $\lambda \geq \kappa$. Thus (κ,λ)-ITP forms a principle that is related to supercompactness the same way the tree property is related to weak compactness. "(ω_2,λ)-ITP holds for all $\lambda \geq \omega_2$" is consistent relative to a supercompact cardinal, and it implies the failure of a weak version of square ($\neg\square_{\lambda,\omega_1}$ for all $\lambda \geq \omega_1$ in particular), so that the best known lower bounds for consistency strength are applicable.

PFA implies (ω_2, λ)-ITP holds for all $\lambda \geq \omega_2$. This can be seen as an affirmation that the consistency strength of PFA really is that of a supercompact. We hope it forms a unified framework that can be utilized under PFA to derive consequences of the "supercompactness" of ω_2 besides the failure of square. For example, we conjecture the principle implies SCH.

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My main interest is iterated forcing; in my diploma thesis I analyzed Shelah's proof of the consistency of "there is no p-point", which was obtained with a countable support iteration.

Shelah introduced the notion of oracle-c.c. forcing. An $\aleph_1$-*oracle* is a sequence $\bar{M} = \langle M_\delta : \delta < \omega_1, \delta \text{ limit}\rangle$, where each M_δ is a countable transitive model of (a large enough portion of) ZFC containing all ordinals up to δ, and for each $A \subseteq \omega_1$, the set $\{\delta : A \cap \delta \in M_\delta\}$ is stationary (in other words, $\bar{M}$ is a diamond sequence). A forcing notion P (with universe ω_1) satisfies the *oracle-c.c.* with respect to the oracle $\bar{M}$ if each set which belongs to M_δ and is predense in $P \restriction \delta$ is predense in P as well (at least if this holds for "a large set of δ's"). Oracle-c.c. is an effective version of the c.c.c., and it is preserved under finite support iterations (like c.c.c.).

The reason for considering the notion of oracle-c.c. is its *omitting type theorem* for Π^1_2-formulas: assume a "real type" is given by ω_1-many Π^1_2-formulas; if there is no real in V realizing this type ("the type is omitted"), and the same holds in the extension by a single Cohen real, then we can find an oracle $\bar{M}$ such that any forcing notion satisfying the oracle-c.c. with respect to $\bar{M}$ will also not add a real realizing this type. In particular, the ground model reals always remain non-meager in the extension by an oracle-c.c. forcing (since this is true for the Cohen extension).

One of Shelah's applications of the method is the following: consistently, all automorphisms of the Boolean algebra $\mathcal{P}(\omega)$/finite are trivial.

In [4] the following question appears: is there a parallel to oracle-c.c., where Cohen forcing (in the assumption of the omitting type theorem) is replaced by, e.g., random forcing. In [5], Shelah developed a general framework for this. The iteration is different from both finite and countable support iteration: finite support iteration introduces Cohen reals, which should be forbidden by any reasonable notion of a "random oracle-c.c.", as there are no Cohen reals in the random extension; countable support iteration would destroy the c.c.c.. The crucial claim is that the iteration in this framework ([5]) behaves well in case of the

ω-limit (i.e., it preserves the "random oracle-c.c."); it seems that a construction involving "free limits" (similar to the free limit in [4, Ch. IX]) can be used here.

There are models of ZFC satisfying the *Borel Conjecture* (e.g. the "Laver model", see [2]); the *Dual Borel Conjecture* (i.e., each strongly meager set is countable) can be obtained by adding many Cohen reals; but this destroys the Borel Conjecture. In [1], Bartoszyński and Shelah show that it is possible to obtain a model of the Dual Borel Conjecture avoiding Cohen reals, using the non-Cohen oracle-c.c. framework from [5]. This is a first step towards a proof of the consistency of "Borel Conjecture and Dual Borel Conjecture", which is still open.

I'm interested in this particular question, and would also like to investigate more generally which properties of forcing notions can be preserved in forcing iterations. I'm also interested in variants of oracle-c.c. for obtaining large continuum (larger than $\aleph_2$), as it is done in [3].

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I am mainly working on cardinal characteristics, relations between them, combinatorial consequences of these relations, and restrictions that ZFC imposes on these constellations. The most recent work [3] is devoted to the definability of some combinatorial objects (e.g., mad families, scales) under certain inequalities between cardinal characteristics. This is an area where independence from the usual axioms of mathematics (the Zermelo Fraenkel axiom system together with the axiom of choice, abbreviated ZFC) often arises. Therefore one of the main parts of the work is to find suitable forcing techniques and possibly develop new ones.

This branch of set theory has two directions.

The *"classical"* part, cardinal characteristics of the continuum, is nowadays a well-developed branch of set theory with applications in many mathematical fields, see [1]. Cardinal characteristics allow for a concise description of the premises beyond ZFC in an independence result: for example, a large part of the most important problems of set-theoretic topology can be (sometimes rather simply) solved under certain (in)equalities between cardinal characteristics, see [4].

Cardinal characteristics of the continuum arise from various questions about critical cardinalities of properties of sets of reals. These properties can have combinatorial, measure-theoretic, topological, or some other nature. One of such questions is: How many compact subspaces are needed to cover the space $\mathbb{R}\setminus\mathbb{Q}$ of irrational numbers? The minimal size of such a cover is denoted by $\mathfrak{d}$. Obviously, $\omega < \mathfrak{d} \leq \mathfrak{c} = 2^{\omega}$.

The study of cardinal characteristics *at arbitrary uncountable cardinals* is the newer direction, whose beginning may be traced back to the work [2] of James Cummings and Saharon Shelah on the global behavior of the bounding and dominating numbers, emerging from the theory of cardinal characteristics at ω. The topic often requires some new techniques and approaches and is still not well understood. By a cardinal characteristic we mean here a class-function $\alpha : \mathrm{Card} \to \mathrm{Card}$, the latter standing for the class of all cardinals. For example, we shall be interested in the function $\mathfrak{d}$ assigning to a regular κ the minimal size $\mathfrak{d}(\kappa)$ of a dominating set of functions $f : \kappa \to \kappa$. Given a cardinal characteristic α, the main points here are: the possible values of $\alpha(\kappa)$; the behavior of the function α on classes of regular cardinals; the consistency strength of the inequality $\alpha(\kappa) > \kappa^+$ for a measurable κ, the internal consistency (=consistency in an inner model) of a specific global behavior of α.

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