Diproche - Automatic Proof Checking for Didactical Applications

Problem statement

Mathematics needs practice. Therefore, there are exercises.

Problem: Feedback comes long after exercises have been finished (typically about a week later) and can in particular not enter the process of solving and presenting a solution.

Approach: Partially automatize the feedback.

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"The third motivation for the QED project is education. (...) The development of mathematical ability is notoriously dependent upon "doing" rather than upon "being told" or "remembering". The QED system will provide, via such techniques as interactive proof checking algorithms (...), an opportunity for the one-on-one presenting, checking, and debugging of mathematical technique, which it is so expensive to provide by the method of one trained mathematician in dialogue with one student. QED can provide an engaging and non-threatening framework for the carrying out of proofs by students, (...)."

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Some sample proofs

A mathematical argument can proceed by a step-by-step deduction, but it doesn't have to:

Example 1: On Monday, 8.00, Gandalf starts in the Shire and travels to Rivendell, where he has a date with Galadriel at 19.00. On Tuesday morning, he starts in Rivendell at 8.00 and travels back to the Shire along the same route. Show that there is a time of the day at which he was in the same spot on Monday and Tuesday.

Solution: Imagine two Gandalfs travelling on the same day. Clearly, they must meet.

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Example 2: Picture Proof for $\Sigma_{i=1}^{n} i = \binom{n+1}{2}$.

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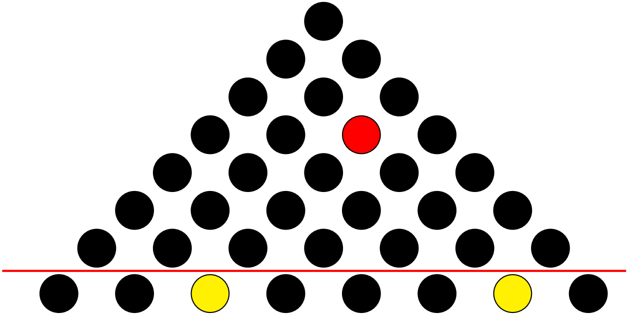

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Example 3: Do objects with a larger weight have a greater fall velocity? Imagine two objects with weight a and $b > a$. Join them with a rope. The compound object should (i) fall quicker because it has weight $a + b$ and (ii) fall slower because the objects with weight b is 'slowed down' by the object with weight a.

An essential part of mathematical understanding and proving consists in conducting thought experiments, looking a the problem from a new perspective etc.

This is an important part of mathematics - in particular of mathematical creativity - that has little to do with logical deduction.

On the other hand: Step-by-step deduction.

Example 4: Show that, for all sets A, B, C, we have $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Proof: "$\subseteq$": Let $x \in A \cap (B \cup C)$. Then we have $x \in A$ and $x \in B \cup C$. Hence, we have $x \in B$ or $x \in C$. If $x \in B$, then $x \in A \cap B$ and hence $x \in (A \cap B) \cup (A \cap C)$. If $x \in C$, then $x \in A \cap C$ and hence $x \in (A \cap B) \cup (A \cap C)$. In any case, we have $x \in (A \cap B) \cup (A \cap C)$.

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A system that takes proof attempts in a controlled form that is still natural for beginner's students, checks it and provides helpful feedback.

Feedback:
(1) Proofreading of natural language, type checking ("Suppose we have $7 + \emptyset$"), logical consistency, achievement check for proof goal.
(2) Hints for proof search: (1) general ("to show $A \to B$, try to assume A and to deduce B"), (2) specific (proof attempt is completed automatically (if possible) and then an intermediate step is proposed) (3) heuristical (hints explictly provided by the person who posed the exercise).
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Naproche (NAtural PROof CHEcking) is a joint project of the logic group in Bonn and linguists from Duisburg-Essen (Bernhard Schröder), initiated by Peter Koepke. The system was essentially developed by Marcos Cramer in his dissertation. Further important contributors are Bernhard Fisseni, Daniel Kühlwein, Nikolay Kolev and others.

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Theorem 9: Fix x, y. Then precisely one of the following cases holds: Case 1: $x = y$. Case 2: There is a u such that $x = y + u$. Case 3: There is a v such that $y = x + v$.
Proof: Fix x, y. By theorem 7, case 1 and case 2 are inconsistent and case 1 and case 3 are inconsistent. Suppose case 2 and case 3 hold. Then $x = y + u = (x + v) + u = x + (v + u) = (v + u) + x$. Contradiction by theorem 7. Thus case 2 and case 3 are inconsistent. Thus for all x, y, at most one of case 1, case 2 and case 3 holds. (...)

Theorem 9: For given x and y, exactly one of the following must be the case: 1) x=y 2) There exists a u such that $y = x + v$ 3) There exists v such that $y = x + v$
Proof: A) By Theorem 7, cases 1) and 2) are incompatible. Similarly, 1) and 3) are incompatible. The incompatibility of 2) and 3) also follows from Theorem 7: for otherwise, we would have
$x = y + u = (x + v) + u = x + (v + u) = (v + u) + x$. Therefore we can have at most one of the cases 1), 2) and 3). (...)

Naproche and Didactics

In spite of its strength, Naproche is not suitable for a didactical application in German universities, due to several reasons:

- The input language is English
- 'too smart': will accept steps that a beginner student should elaborate. (In many cases, it will accept the empty string as a sufficient proof for an exercise problem.)
- No possibility to control the admissible deduction rules; but teaching should start with demanding very basic steps and then gradually allow for greater leaps.
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- Not taylored to the language and prerequisites of beginner's exercises; e.g., no use of $\rightarrow$ for 'I am now showing one direction of an equivalence', no module for term manipulations or calculations such as $0 \leq (a-b)^2 = (a-b)(a-b) = (a^2 - ab - ba + b^2) = (a^2 - 2ab + b^2)$.

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- Preprocessing of the input string
- Annotation
- Determination of text structure
- Generation of ATP-tasks
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Example

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list format: [[es,gelte,a],[ferner,gelte,[a,− >,b]],[dann,folgt,b]]

Annotated Format:
[[1,[],[],ann,bam,[]],[2,[a],[],ang,[],a],[3,[a,b],[],ang,[],[a,− >,b]],
[4,[b],[],beh,[],b],[5,[],[],ann,bem,[]]]

text structure graph: [[2,4],[3,4]] - encodes which assumptions are available at which text portions.

ATP-task (for "line" 4): [[a,[a,− >,b]],[b]]

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Example

Input: "Es gelte a. Ferner gelte $(a \to b)$. Dann folgt b."

list format: [[es,gelte,a],[ferner,gelte,[a,$-$ $>$,b]],[dann,folgt,b]]

Annotated Format:
[[1,[],[],ann,bam,[]],[2,[a],[],ang,[],a],[3,[a,b],[],ang,[],[a,$-$ $>$,b]],
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text structure graph: [[2,4],[3,4]] - encodes which assumptions are available at which text portions.

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The controlled ATP

The ATP is specifically designed to only accept proof steps that should be accepted as sufficient in the context of beginner's exercises.

It is thus intentionally weak: It only accepts a step when it matches one of the (many) rules in its database, i.e. there is no iteration in the proof search.

In addition, one can specify 'difficult degrees', i.e. subsets of prover rules, for specific exercises. In this way, the ATP can become more liberal as the student advances.

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Assumptions: “Es gelte/Angenommen, es gilt/Nehmen wir an, dass/...”

Claims: “Dann folgt/Es gilt/Damit haben wir/...”

Annotations: "Beweis:", "=>", "<=", "durch widerspruch", "wir zeigen, dass...", "qed", "Fall 1", paragraphs

An assumption is valid (only) in the paragraph in which it is introduced.
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Wir zeigen $((a \vee b) \leftrightarrow ((a \rightarrow b) \rightarrow b))$.
$=>$ Angenommen, es gilt $(a \vee b)$. Angenommen ferner, es gilt $(a \rightarrow b)$. Falls a gilt, so gilt b. Falls b gilt, so gilt ebenfalls b. Also gilt b.
Damit folgt $((a \rightarrow b) \rightarrow b)$.
qed.
$<=$
Angenommen, es gilt $((a \rightarrow b) \rightarrow b)$. Nehmen wir an, es gilt $\neg a$.
Dann gilt auch $(a \rightarrow b)$. Damit folgt b.
Also folgt $(\neg a \rightarrow b)$. Damit folgt $(a \vee b)$.
qed.
Also gilt auch $(((a \rightarrow b) \rightarrow b) \rightarrow (avb))$.
Damit folgt nun endlich $((a \vee b) \leftrightarrow ((a \rightarrow b) \rightarrow b))$.
Qed.

Sample text: propositional logic

Wir zeigen $((a \vee b) \leftrightarrow ((a \rightarrow b) \rightarrow b))$.
=> Angenommen, es gilt $(a \vee b)$. Angenommen ferner, es gilt $(a \rightarrow b)$. Falls a gilt, so gilt b. Falls b gilt, so gilt ebenfalls b. Also gilt b.
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qed.
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Sample text: Boolean set theory

Es seien A, B, C und D Mengen. Es gelte $(A \cap B) = \emptyset$. Ferner gelte $(C \subseteq A)$ und $(D \subseteq B)$. Wir zeigen $(C \cap D) = \emptyset$.

Beweis.

Es gelte $x \in (C \cap D)$. Dann folgt $(x \in C)$. Also folgt $(x \in A)$. Ferner gilt $(x \in D)$. Damit gilt auch $(x \in B)$. Damit haben wir $(x \in (A \cap B))$.
Also gilt $(x \in \emptyset)$. Widerspruch.

Also gilt $(C \cap D) = \emptyset$.

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Also gilt $(x \in \emptyset)$. Widerspruch.

Also gilt $(C \cap D) = \emptyset$.

qed.

Sample text: even/odd

Es sei n eine natuerliche Zahl. Wir zeigen $n(n+1)$ ist gerade.

Beweis.

Angenommen, n ist gerade. Dann ist auch $n(n+1)$ gerade.

Nehmen wir nun an, n ist ungerade. Dann ist $(n+1)$ gerade. Damit ist $n(n+1)$ ebenfalls gerade.

Also ist $n(n+1)$ gerade.

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Also ist $n(n+1)$ gerade.

qed.

Sample text: Induction, Gauß sum

Es gilt $\Sigma_{i=1}^{1} i = 1 = \frac{1\cdot(1+1)}{2}$.
Es gelte $\Sigma_{i=1}^{n} i = \frac{n(n+1)}{2}$.
Dann folgt $\Sigma_{i=1}^{n+1} i = \Sigma_{i=1}^{n} i + (n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2} = \frac{(n+1)((n+1)+1)}{2}$.
Also gilt $\Sigma_{i=1}^{n+1} i = \frac{(n+1)((n+1)+1)}{2}$.
Also haben wir $\Sigma_{i=1}^{n} i = \frac{n(n+1)}{2} \to \Sigma_{i=1}^{n+1} i = \frac{(n+1)((n+1)+1)}{2}$.
Damit folgt $\forall n \Sigma_{i=1}^{n} i = \frac{n(n+1)}{2} \to \Sigma_{i=1}^{n+1} i = \frac{(n+1)((n+1)+1)}{2}$.
Induktiv folgt nun $\forall n \Sigma_{i=1}^{n} i = \frac{n(n+1)}{2}$.
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Damit folgt $\forall n \Sigma_{i=1}^{n} i = \frac{n(n+1)}{2} \to \Sigma_{i=1}^{n+1} i = \frac{(n+1)((n+1)+1)}{2}$.
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Goaltracing

In Diproche, one can declare proof goals through formulations such as:

Wir zeigen: Für alle Mengen A, B mit $A \subseteq B$ gilt $(A \cup B) = B$.

The goal tracing then attempts to determine the current goal at each position in the proof test.
For example, the annotation "$\Rightarrow$" has the effect that the current proof goal is changed from $(\phi \leftrightarrow \psi$ to $(\phi \rightarrow \psi)$
It is then checked for every proof ending marker whether the corresponding goal has indeed been reached. The goal only counts as reached when it is explicitly stated as the final statement before the endmarker.

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Type-Checking

Many beginner's mistakes happen at the type level. Either objects are used in a way they cannot ('Assume $7 + \emptyset$') or they are used without having been declared at all.
This is taken care of by a type-checking algorithm. Declaration can be made by formulations such as

"Es seien a, b und c natuerliche Zahlen und d eine reelle Zahl'."

In particular, it is checked whether the content of an assumption or a claim is indeed a proposition.

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Fallacy diagnosis: The Anti-ATP

A good tutor cannot just tell whether a step is right or wrong, but only have an educated guess at what's behind a mistake.

On the one hand, this allows one to specifically address misconceptions, and on the other hand, one can recognize and encourage possibly good ideas in a proof attempt.

At least the first function can be realized in Diproche to a certain extent.

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The Anti-ATP works like an ATP, but has typical formal fallacies instead of sound proof rules, such as.

- From $A \vee B$ and A, derive $\neg B$
- From $A \rightarrow B$ and $\neg A$, derive $\neg B$
- From $\neg A$, derive $\neg(A \rightarrow B)$
- From $\neg(A \wedge B)$, derive $\neg A \wedge \neg B$

Each of these fallacies has an internal index. When the ATP cannot verify a step, it is passed on to the anti-ATP. When the proof step in question is realizable by a formal fallacy, an according feedback is given.

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The anti-ATP has a subroutine for false term manipluations, such as:

- $\frac{n^2}{n^3} = \frac{2}{3}$
- $\frac{n^2}{n^4} = \frac{n}{n^2}$
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The system provides three kinds of hints that users can request when they are stuck at a certain point in a proof:

- Prescribed, problem-specific hints. These are entered by the teacher by hand and resemble those hints that are typically found in the 'solution' part of a textbook.
- General strategical hints, either based on the goal ("In order to prove $A \to B$, assume A and try to prove B") or based on the assumptions ("When you have a disjunction in your assumptions, try a case distinction").
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Problem Database

For each area covered by Diproche, one can enter problems.

A problem consists of:

- An index.
- A verbal formulation for the user.
- A formalization of the proof goal.
- A specification of the degree of difficulty (=set of prover rules).
- A list of assumptions one may use during the solution.
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In order to increase the available training material, it is desirable to generate practice problems automatically. Currently, there are routines for automatically generating problems in the following areas:

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Mathematical work takes place in a number of contexts, each of which has its own...

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- Refine the hints
- Extend the problem generators
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