For this lesson there should be a projector!

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Sample Lesson Plan 2

0 min

We already know how to transform expressions/terms by adding, subtracting and multiplying polynomials. Now we will learn how to factor polynomials, which is the inverse operation to multiplying polynomials. We will use some of the examples from the textbook, for practice it is recommended that at home you go through all examples in the textbook by yourself.

First examples of Factoring:

Find common factor and factor out:

$10x + 30$

$= 10 \times x + 10 \times 3$

$= 10(x + 3)$

REMARK:
Also $10x + 30 = 5(2x + 6)$, normally factor out the greatest common factor.

5 min

Find common factor and factor out:

$4xy + 3x + 20x^2$

$= x \times 4y + x \times 3 + x \times 20x$

$= x(4y + 3 + 20x)$

REMARK:
If we factor out variables, normally factor out the highest degree of that variable.

Find common factor and factor out:

$5x^4 + 20x^3 = 5x^3(x + 4)$

$5x^3(x + 4)$

$= 5x^3 \times x + 5x^3 \times 4$

$= 5x^4 + 20x^3$

REMARK:
Check, at least mentally, your result by multiplying.

10 min

Find common factor and factor out:

$-4x^2 + 2x - 8y$

$= (-2) \times 2x^2 - (-2) \times x + (-2) \times 4y$

$= (-2)(2x^2 - x + 4y$

REMARK:
When factoring out a negative number do not forget to "switch" the signs.

Find common factor and factor out:

$4x^3 - 15 + 20x^2 - 3x$

$= \underline{4x^3 + 20x^2} - 3x - 15$

$= 4x^2(x + 5) + \underline{(-3)x + (-3)5}$

$= 4x^2(x + 5) + (-3)(x + 5)$

$= (x + 5)(4x^2 - 3)$

REMARK:
Sometimes it is necessary to rearrange the polynomial to see the groups and the common factors.
To factor out a common binomial one sometimes has to group the terms and factor more than once.

15 min

How to Factor out:

If term can be written in the form "$a \times b + a \times c$" then we can factor out a and get
$a \times b + a \times c = a(b + c)$.
Here a, b, and c can also be terms.
Check your result by multiplying.

How to factor Trinomials :

We now will introduce a method to factor trinomials, i.e. polynomials of the type $ax^2 + bx + c$.
Remember that we multiplied two binomials using the FOIL method:

$$(2x+3)(4x+5) = 2x \times 4x + 2x \times 5 + 3 \times 4x + 3 \times 5 = (2 \times 4)x^2 + (2 \times 5 + 4 \times 3)x + (3 \times 5)$$

The general rule is

$$(rx+p)(sx+q) = rsx^2 + (sp+rq)x + (p \times q).$$

Use a Projector or a special part of the BB for this

20 min

The method to factor a trinominal:

If we have a trinominal of type $ax^2 + bx + c$, we first see if a,b, and c have a common factor and factor that out. Also if a is negative, we factor out -1.

Assume we have a polynomial of the form $ax^2 + bx + c$, where a,b, and c have no common factor and a is positive.

1) We find pairs of factors r and s so that $a = rs$.
2) We find pairs of factors p and q so that $c = pq$.
2) We test those factor pairs to find if there is a combination that gives $sp + rq = b$.
3) If there is such a combination of p, q and r, s, we have
   $ax^2 + bx + c = (rx+p)(sx+q)$.

Find common factor and factor out:

$3x^2 + 10x + 8$

pair $r \times s = a$	pair $p \times q = c$	sum $sp + rq$
1×3	1×8	$3 \times 1 + 1 \times 8 = 11$
1×3	8×1	$3 \times 8 + 1 \times 1 = 25$
$\underline{1 \times 3}$	$\underline{2 \times 4}$	$\underline{3 \times 2 + 1 \times 4 = 10}$
1×3	4×2	$3 \times 4 + 1 \times 2 = 14$

So $3x^2 + 10x + 8 = (x+2)(3x+4)$

REMARK:
Check increasing factors of a and c,
start with $1 \times a$ and $1 \times c$.
Do all possible combinations!
The order of the variables p and q is very important!

25 min

Find common factor and factor out:

$3x^2 - 10x + 8$

pair $r \times s = a$	pair $p \times q = c$	sum $sp + rq$
1×3	$(-1) \times (-8)$	$3 \times (-1) + 1 \times (-8) = -11$
1×3	$(-8) \times (-1)$	$3 \times (-8) + 1 \times (-1) = -25$
$\underline{1 \times 3}$	$\underline{(-2) \times (-4)}$	$\underline{3 \times (-2) + 1 \times (-4) = -10}$
1×3	$(-4) \times (-2)$	$3 \times (-4) + 1 \times (-2) = -14$

So $3x^2 - 10x + 8 = (x-2)(3x-4)$

REMARK:
If b is negative and c is positive, we have to look at the negative factors of c. Remember "minus times minus equals plus".

30 min

Find common factor and factor out:

$3x^2 + 10x - 8$

pair $r \times s = a$	pair $p \times q = c$	sum $sp + rq$
1×3	$(-1) \times 8$	$3 \times (-1) + 1 \times 8 = 5$
1×3	$1 \times (-8)$	$3 \times 1 + 1 \times (-8) = -5$
1×3	$(-8) \times 1$	$3 \times (-8) + 1 \times 1 = -23$
1×3	$8 \times (-1)$	$3 \times 8 + 1 \times (-1) = 23$
1×3	$(-2) \times 4$	$3 \times (-2) + 1 \times 4 = -2$
1×3	$2 \times (-4)$	$3 \times 2 + 1 \times (-4) = 2$
1×3	$(-4) \times 2$	$3 \times (-4) + 1 \times 2 = -10$
$\underline{1 \times 3}$	$\underline{4 \times (-2)}$	$\underline{3 \times 4 + 1 \times (-2) = 10}$

So $3x^2 + 10x - 8 = (x + 4)(3x - 2)$

REMARK:
If c is negative, we have to check all combinations of factors of c with one negative and one positive factor.
Watch out for the sign of the sum, it depends on which factor is negative!

Tips for Factoring $ax^2 + bx + c$:

1. If necessary, rewrite the trinominal in descending order. Factor out common factors, for the rest of the tips we will assume that a, b, and c have no common factor and that a is positive.
2. Make a table with three columns: factor pairs r, s of a, factor pairs p, q of a and the sum $sp + rq$. Check if the sum equals b. Remember:
   - Start with the "smallest" (like $1 \times a$) factors for a and b and for every factorization of a work through all factorizations of c, remember that the order of the factors p,q of c is important.
   - If c is positive and b is positive, you only have to check the positive factors of c.
   - If c is positive and b is negative, you only have to check the negative factors of c.
   - If c is negative, you have to check all combinations of factors of c with one negative and one positive factor. Watch out for the sign of the sum.
3. Check your result by multiplying the binomials.

40 min

How to factor (simpler) Trinomials:

If we have a trinomial of the type $x^2 + bx + c$, things get a little easier. Since here a is 1, it has only the factors 1,1, so we do not need the "pair r,s" column.

Remark:
If we have a trinominal of type $ax^2 + bx + c$ we first see if a is a common factor of a,b, and c. If it is we can factor a out and get the simpler form.

Use projector or leave on BB

Assume we have a polynomial of the form $x^2 + bx + c$.

1) We find pairs of factors p and q so that $c = pq$.
2) We test those factor pairs to find if there is one such that $b = p + q$.
3) If there is such a pair p, q of factors of c, we have

   $x^2 + bx + c = (x + p)(x + q)$.

Find common factor and factor out:

$x^2 + 9x + 8$

pair $p \times q = c$	sum $p + q$
$\underline{1 \times 8}$	$\underline{1 + 8 = 9}$
2×4	$2 + 4 = 6$

So $x^2 + 9x + 8 = (x + 1)(x + 8)$

REMARK:
Check increasing factors of c, start with 1!

Find common factor and factor out:

$y^2 - 9y + 20$

pair $p \times q = c$	sum $p + q$
$(-1) \times (-20)$	$-1 - 20 = -21$
$(-2) \times (-10)$	$-2 - 10 = -12$
$\underline{(-4) \times (-5)}$	$\underline{-4 - 5 = -9}$

So $y^2 - 9y + 20 = (y - 4)(y - 5)$

REMARK:
If b is negative and c is positive, we have to look at the negative factors of c. Remember "minus times minus equals plus".

45 min

Find common factor and factor out:

$x^3 - x^2 - 30x$

$= x(x^2 - x - 30)$

So we work with $x^2 - x - 30$:

pair $p \times q = c$	sum $p + q$
$(-1) \times 30$	$-1 + 30 = 29$
$1 \times (-30)$	$1 - 30 = -29$
$(-2) \times 15$	$-2 + 15 = 13$
$2 \times (-15)$	$2 - 15 = -13$
$(-3) \times 10$	$-3 + 10 = 7$
$3 \times (-10)$	$3 - 10 = -7$
$(-5) \times 6$	$-5 + 6 = 1$
$\underline{5 \times (-6)}$	$\underline{5 - 6 = -1}$

REMARK:
Do not forget to factor out common factors before trying to factor the trinomial.
If c is negative we have to consider all combinations of positive and negative factors of c.

So $x^2 - x - 30 = (x + 5)(x - 6)$, which means $x^3 - x^2 - 30x = x(x^2 - x - 30) = x(x + 5)(x - 6)$.

Tips for Factoring $x^2 + bx + c$:

1. If necessary, rewrite the trinominal in descending order. Make a table of all factor pairs of c and their sums. Check if two factors add up to b. Remember:
   - If c is positive and b is positive, you only have to check the positive factors of c.
   - If c is positive and b is negative, you only have to check the negative factors of c.
   - If c is negative, you have to check all combinations of factors of c with one negative and one positive factor. Watch out for the sign of the sum.
2. Check your result by multiplying the binomials.

50 min

Homework (from Exercise Set 5.3): 2), 4), 8), 14), 18), 22), 26), 29), 32), 35), 38), 44).
Grade Problems 14), 18) and 32).
Homework (from Exercise Set 5.4): 2), 6), 12), 18), 24), 28), 57), 48), 79).
Grade Problems 2), 18) and 48).