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# Sample Lesson Plan 1

We already know how to solve quadratic equations using the method of factoring. But since one has to guess the right number when using this method it is not always successful. So today we will work our way to an alternative method for solving quadratic equations, one that can be used always.
We will use some of the examples from the textbook, it is recommended that at home you go through those examples by yourself again for practice.

**First examples of easy Squares:**

*5min*

Solve the following equations:

$x^2 = 1$

$x^2 = 9$

$x^2 = 36$

REMARK:
First obtain the positive solution, then show that there is also a negative solution
Introduce the notation $x = \pm a$:

$x = \pm a$

means $x = +a$ **or** $x = -a$

**The Principle of Square Roots:**
If $x^2 = a$ then the solution is $x = \pm\sqrt{a}$,
i.e. $x = \sqrt{a}$ **or** $x = -\sqrt{a}$.

*10min*

Solve the following equations:

$2x^2 = 50$

$7x^2 = 5$

REMARK:
Remind the students about multiplying equations (by fractions):

$b \times x^2 = a \ \ | \div b$

$x^2 = \frac{a}{b}$

means $x = +\sqrt{\frac{a}{b}}$ **or** $x = -\sqrt{\frac{a}{b}}$

Solve the following equations:

$5x^2 - 125 = 0$

$x^2 + 9 = 0$

REMARK:
Remind the students about adding\subtracting to\from an equation:

$x^2 - a = b \ \ | + a$

$x^2 = b + a$

means $x = +\sqrt{a+b}$ **or** $x = -\sqrt{a+b}$

Also remind them about complex solutions:

$x^2 = -9 \Rightarrow x = \pm\sqrt{-9} = \pm 3\sqrt{-1} = \pm 3i$

*15min*

Solve the following equation:

$\underline{(x-2)^2 = 25 \ :}$

$\sqrt{(x-2)^2} = \pm\sqrt{25}$

$x - 2 = \pm\sqrt{25},$

which means
$x - 2 = +5$ **or** $x - 2 = -5$,
so $x = 7$ **or** $x = -3$

REMARK:
Show them by checking the solution that this really works. And remark that variables can always denote terms.

**The Principle of Square Roots revisited:**
If $(\text{term})^2 = a$ then the solution is $(\text{term}) = \pm\sqrt{a}$,
i.e. $(\text{term}) = \sqrt{a}$ or $(\text{term}) = -\sqrt{a}$.

*20min*

Solve the following equation:

$\underline{x^2 + 6x + 9 = 2:}$

$(x+3)^2 = 2$

$x + 3 = \pm\sqrt{2}$

which means
$x + 3 = \sqrt{2}$ **or** $x + 3 = -\sqrt{2}$,
so $x = -3 + \sqrt{2}$ **or** $x = -3 - \sqrt{2}$.

REMARK:
Remind Students about binominals:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

*25min*

**Completing the Square:**
We will now use the revised Principle of Square Roots to find a method that will enable us to solve every quadratic equation.

*Clear BB, use left side*

Solve the following equation:

$\underline{x^2 + 6x - 16 = 0:}$

$x^2 + 6x - 16 = 0 \mid +16$

$x^2 + 6x \quad = 16 \mid +9 = \left(\frac{6}{2}\right)^2$

$x^2 + 6x + 9 = 16 + 9$

$x^2 + 6x + 9 = 25 \mid$ Use binomial formula

$(x+3)^2 = 25 \mid$ Use Principle of Square Roots

$x + 3 = \pm\sqrt{25},$

which means
$x + 3 = 5$ **or** $x + 3 = -5$,
so $x = 2$ **or** $x = -8$.

REMARK:
Explain what you do, remind the Students about the binomial formula and that $(\frac{6}{2})^2 = 3^2 = 9$, state where you use the Square Root Principle.

*30min*

## How to solve a Quadratic Equation in $x$ by Completing the Square:

*Keep this on the BB*
*Use middle or right*

1. Isolate the terms with variables on one side of the equation, and arrange them in descending order.
2. Divide both sides by the coefficient of $x^2$.
3. Complete the square by taking half of the coefficient of $x$ and adding its square to both sides.
4. Express one side as the square of a binominal and simplify the other side.
5. Use the principle of square roots.
6. Solve for $x$ by adding or subtracting on both sides.

*35min*

Solve the following equation:

$\underline{3x^2+7x-2=0:}$

$3x^2+7x-2=0$ Step 1: $+2$

$3x^2+7x \quad =2$ Step 2: $\div 3$

$x^2+\frac{7}{3}x \quad =\frac{2}{3}$ Step 3: $+(\frac{1}{2}\times\frac{7}{3})^2=(\frac{7}{6})^2)=\frac{49}{36}$

*Use right or clear left*

$x^2+\frac{7}{3}x+\frac{49}{36}=\frac{2}{3}+\frac{49}{36}$ Step 4: factor and simplify $(\frac{2}{3}=\frac{24}{36})$

$\left(x+\frac{7}{6}\right)^2=\frac{73}{36}$ Step 5: Principle of square roots

$x+\frac{7}{6}=\pm\sqrt{\frac{73}{36}},$ which means in Step 6 we get:

$$x=-\frac{7}{6}+\sqrt{\frac{73}{36}} \textbf{ or } x=-\frac{7}{6}-\sqrt{\frac{73}{36}},$$
$$\text{so } x=\frac{-7+\sqrt{73}}{6} \textbf{ or } x=\frac{-7-\sqrt{73}}{6}.$$

REMARK:
Go through Step by Step, explain your calculations. Say that it is Example 11 from the Textbook.

*40min*

Solve the following equation:

$\underline{x^2-6x+4=0:}$

$x^2-6x+4=0$ Step 1: $+4$

$x^2-6x \quad =-4$ Step 2: coefficient of $x^2$ is 1

*Do only if enough time*

$x^2-6x \quad =-4$ Step 3: $+(\frac{6}{2})^2=3^2=9$

$x^2-6x+9=-4+9$ Step 4: factor and simplify

$(x-3)^2=5$ Step 5: Principle of square roots

$x-3=\pm\sqrt{5},$ which means in Step 6 we get:

$x=3+\sqrt{5}$ **or** $x=3-\sqrt{5}.$

REMARK:
Remind the Students about the other binomial formula.

Homework (from Exercise Set 8.1): 2), 15), 23), 28), 35), 40), 45), 46), 49), 55), 56), 61), 70).

*50min* *Grade Problems 28), 40) and 56).*