Working group: Complex geometry

Rational curves on Fanos and Lagrangian fibrations

The ultimate goal of this working group is to understand Hwang's proof of the conjecture that the base space of any Lagrangian fibration of a holomorphic symplectic manifold is the projective space $\mathbb{P}^n$. Hwang proves this under the assumption of smoothness of the base space in [2]. At the same time, the working group will be the occasion to learn some of the very useful techniques using rational curves of minimal degree to study Fano manifolds.

Suppose X is a projective irreducible holomorphic symplectic manifold, i.e. X is a simply-connected projective manifold which admits a unique (up to scaling) $\sigma \in H^0(X, \Omega_X^2)$ which is everywhere non-degenerate. It has been conjectured that after deformation any such variety admits a Lagrangian fibration $X \longrightarrow \mathbb{P}^n$. Due to work of Matsushita [9, 10] one knows that any non-constant morphism with positive fibre dimension $X \longrightarrow B$ is a Lagrangian fibration with the generic fibre being a complex torus. Matsushita has also obtained certain restrictions on the base manifold B, which suggested $B \simeq \mathbb{P}^n$. (Note that the first part of the program, namely the existence of a deformation that admits a Lagrangian fibration over whatever base is still wide open.)

However, holomorphic symplectic manifolds and the conjecture will only be in the background and we will study rational curves on Fano manifolds of Picard rank one for most of the time. Rational curves of minimal degree have been important in the theory of Kollar et al and Campana on rational connected varieties. The main technical tool designed by Hwang and Mok to study varieties with many rational curves is the *minimal rational tangent variety*, roughly the variety of tangent directions of these rational curves passing through a fixed very general point.

The plan is to get prepared for a **talk of Jun-Muk Hwang on** [2] **on May 27** and to go into the details of his proof after this.

Preliminary plan for talks

0. Introduction, Work of Matsushita (Date 8.4.)
1. Reminder of deformation theory (use [8] and [11]). Definition of the variety of minimal rational tangents and first properties (see Sections 1.1-1.3 in [6]). (Date: 15.4.)
2. Explicit examples (Section 1.4 in [2]) (Date: 22.4.)
3. Gauss maps, Cartan-Fubini extension (see [4], Section 3 in [2], see also [7, Sect. 3] (Dates: 29.4., 6.5.)
4. Section 2 in [6] (combine with [1], [5]).
5. **J.-M. Hwang** will give a talk introducing us to the proof in [2]. (Date: 27.5.)
6. Strategy of [2] (Date 3.6.)
7. Proof (Dates: 10.6., 17.6.)

(Dates are provisional, except for 5.)

References

[1] Carolina Arajou, *Rational curves of minimal degree and characterization of projective space*, Math. Ann. 335 (2006).

[2] Jun-Muk Hwang, *Base manifolds for fibrations of projective irreducible symplectic manifolds*, 0711.3224.

[3] Jun-Muk Hwang, *Rigidity of rational homogeneous spaces*, Proc. ICM Madrid.

[4] Jun-Muk Hwang, Ngaiming Mok, *Birationality of he tangent map for minimal rational curves*, Asian J. Math. 8 (2004).

[5] Jun-Muk Hwang, *Deformations of holomorphic maps onto Fano manifolds of second and fourth Betti numbers 1*, Ann. Inst. Fourier 57 (2007).

[6] Jun-Muk Hwang, *Geometry of Minimial Rational Curves on Fano Manifolds*, ICTP lecture notes (2000) available at http://publications.ictp.it/

[7] Stefan Kebekus, *Families of singular rational curves*, J. Alg. Geom. 11 (2002).

[8] Janos Kollar, *Rational curves on algebraic varieties*, Erg. Math. (Springer).

[9] Daisuke Matsushita, *On fibre space structures of a projective irreducible symplectic manifold*, Top. 38 (1999).

[10] Daisuke Matsushita, *Higher direct images of Lagrangian fibrations*, Amer. J. Math. 127 (2005).

[11] Olivier Debarre, *Higher dimensional algebraic geometry*, Universitext (Springer).

The working group takes place on Tuesday, 12:15-14:00. SR A. Beringstr. 4. For further information please contact Daniel Huybrechts (huybrech@math.uni-bonn.de) or Emanuele Macrì (macri@math.uni-bonn.de).

The first talk will be on April 8.