SEMINAR ON COMPLEX GEOMETRY

The seminar will cover parts of the theory contained in [1, 2, 5]. (The references [3, 4, 6] are more advanced and can be used to go deeper into the subject.)

The seminar takes place on Thursdays, 14-16, in Seminar room 0.011 during the summer term 2013. If you are interested in participating, please send an e-mail listing two to three talks you would want to give to either huybrech@math.uni-bonn.de or schuerg@math.uni-bonn.de. The deadline for registering is **February 28**. We will assign the talks by March 8.

Prerequisites:

- You should be well acquainted with the local theory of holomorphic functions of several variables as presented in Section 1.1 of [5]. In particular, you should be familiar with Hartogs' Theorem and the Weierstrass Preparation Theorem.
- A guiding example throughout the seminar will be projective space equipped with the Fubini-Study metric. Feeling comfortable with this example as early as possible will be of great help.

All references in the following are to [5].

1. **Complex Manifolds and Holomorphic Vector Bundles (11.4.)**
Assigned Reading: Sections 2.1 and 2.2.
Talk: Prove Siegel's Theorem. Exponential Sequence and first Chern class. Normal bundle sequence and the adjunction formula.

2. **Differential Forms on Complex Manifolds (18.4.)**
Assigned Reading: pp. 25–28. Sections 1.3 and 2.6.
Talk: Prove 1.3.7 and 1.3.8. Prove Prop. 2.6.11. Define the Dolbeaut complex and Dolbeaut cohomology groups. Prove Cor. 2.6.21.

3. **Kähler Manifolds (25.4.)**
Assigned Reading: pp. 28–29, pp. 48–49, pp. 116–120.
Talk: Introduce the Fubini-Study metric in detail (Example 3.1.9). Prove $\int_{\mathbb{P}^1} \omega_{\mathrm{FS}} = 1$. Prove 3.1.10, 3.1.11.

4. **Hermitian Linear Algebra (2.5.)**
Assigned Reading: Section 1.2.
Talk: Recall the definition of the Lefschetz operator and its dual. Prove that they define an $\mathfrak{sl}_2$-representation on $\bigwedge^* V^*$ (1.2.26) and the Lefschetz decomposition theorem (1.2.30).

5. **Kähler Identities (16.5.)**
Assigned Reading: Section 3.1.
Talk: Give an overview over the operators occuring in the Kähler identities (3.1.12) and prove the identities.

6. **Hodge Decomposition (6.6.)**
Assigned Reading: Section 3.2.
Talk: Prove Proposition 3.2.6 and Corollary 3.2.12. Show how remark 3.2.7 implies the corresponding symmetries of the cohomology groups. Explain the diagram on p. 138.

7. **Lefschetz Theorems (13.6.)**
Assigned Reading: Section 3.3.
Talk: Prove the Lefschetz $(1,1)$-theorem (Lemma 3.3.1, Prop. 3.2.2). Derive the Hard Lefschetz theorem (3.3.13) from the decomposition of the exterior algebra.

8. **Hodge Theorems (20.6.)**
Assigned Reading: Section 3.3.
Talk: Prove the Hodge Index Theorem (3.3.16) and the signature formula (3.3.17). This talk is slightly shorter and should be finished in one hour.

9. **Connections and Curvature (27.6.)**
Assigned Reading: Sections 4.2 and 4.3.
Talk: Introduce the Chern connection (Prop. 4.2.14). Discuss Example 4.2.16 ii). Explain the difference to a holomorphic connection. Prove Prop. 4.2.19. Explain the relationship between the curvature of the Chern connection and the Atiyah class (Prop. 4.3.10). Explain Example 4.3.12. This talk is slightly longer and should start already in the previous session.

10. **Chern–Weil theory and Hirzebruch–Riemann–Roch (4.7.)**
Assigned Reading: Sections 4.4 and 5.1.
Talk: Introduce Chern classes, Chern characters and Todd classes (p. 196). Discuss Ex. 4.4.11. Prove Exercise 4.4.7. Explain the statement of the Hirzebruch–Riemann–Roch theorem and the examples on p.233.

11. **Kodaira Vanishing and Weak Lefschetz (11.7.)**
Assigned Reading: Section 5.2.
Talk: Prove Kodaira Vanishing, leaving out the proof of Lemma 5.2.3. Discuss Example 5.2.5. Prove the Weak Lefschetz Theorem.

12. **Kodaira Embedding (18.7.)**
Assigned Reading: Sections 2.5 and 5.3.
Talk: Explain how a complete linear system induces a closed embedding iff it seperates points and tangent vectors. Prove the Kodaira embedding theorem.

References

1. W. Ballmann, *Lectures on Kähler manifolds*, ESI Lectures in Mathematics and Physics, EMS, 2006.
2. M. A. de Cataldo, *The hodge theory of projective manifolds*, Imperial College Press, 2007.
3. J.-P. Demailly, *Complex differential geometry*, `http://www-fourier.ujf-grenoble.fr/~demailly/manuscripts/agbook.pdf`.
4. P. Griffiths and J. Harris, *Principles of algebraic geometry*, Wiley, 1978.
5. D. Huybrechts, *Complex geometry*, Springer, 2004.
6. C. Voisin, *Hodge theory and complex algebraic geometry. I.*, Cambridge University Press, 2002.