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Wintersemester 2010/11

8. Übung Globale Analysis I

Abgabe am Montag, den 13. Dezember, in der Vorlesungspause.

Bei Fehlern oder Fragen bitte eine eMail an: `brief`α`fabianmeier.de` .

Question 1 (exact sequence). (3 points)

Let V be a vector space a $\xi \in V$ be a non-vanishing vector. Then

$$\Lambda_p(V) \xrightarrow{\xi\wedge} \Lambda_{p+1}(V) \xrightarrow{\xi\wedge} \Lambda_{p+2}(V)$$

is an exact sequence, i.e. the image of the first map equals the kernel of the second one.

Question 2 (Star-shaped areas). (5 points)

Let $U \subset \mathbb{R}^n$ be a star-shaped area. Let $f_i \in C^\infty(U)$, $i = 1, ..., n$. Show: There is a $g \in C^\infty(U)$ with

$$\frac{\partial g}{\partial x_i} = f_i, \quad i = 1, ..., n,$$

if and only if the integrability condition

$$\frac{\partial f_i}{\partial x_j} = \frac{\partial f_j}{\partial x_i}, \quad i, j = 1, ..., n,$$

holds.

Question 3 (Lie derivative). (5 points)

For $X \in \mathcal{V}(M)$ let L_X be the Lie derivative and $\iota(X)$ the inner multiplication on $\Lambda^*(M)$. Prove the following equation:

$$L_X = \iota(X) \circ d + d \circ \iota(X)$$

Hint: First show that both expressions are derivations. Then use the fact that both expressions commute with d to reduce the claimed formula to the case of functions.

Question 4 (Lie derivative: explicit). (4 points)

1. Consider the 2-torus T^2 (see last example sheet). Is there a vector field X so that $L_X(dx) = dy$?
2. Can you construct a vector field X on $\mathbb{R}^2$ with $L_X(dx) = dy$?
3. Let α be a closed one-form with $[\alpha] \neq 0$ on a manifold M. Is there a vector field X with $L_X(\alpha) = \alpha$?