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Wintersemester 2010/11

6. Übung Globale Analysis I

Abgabe am Montag, den 29. November, in der Vorlesungspause.

Bei Fehlern oder Fragen bitte eine eMail an: `brief`α`fabianmeier.de` .

Question 1 (Completeness of vector fields). (4 points)

1. Define a smooth vector field on $\mathbb{R}$ which is not complete.
2. Let V be a smooth vector field on $\mathbb{R}$ with $\lim_{x\to\pm\infty} V(x) = 0$ and $\lim_{x\to\pm\infty} \frac{\partial^n V(x)}{\partial^n x} = 0$ for all $n \geq 1$. Show that V is complete.

Question 2 (Proper maps). (5 points)

Let $k \leq n$, $U \subset \mathbb{R}^k$ be open and $f\colon U \to \mathbb{R}^n$ be a submanifold. Furthermore, let f be a *proper* map, i.e. for all compact subsets $K \subset \mathbb{R}^n$ we know that $f^{-1}(K)$ is also compact. Show that f is an embedding.

Question 3 (Flows don't commute). (4 points)

Define the following vector fields in the plane

$$V = x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x}, \qquad W = x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}$$

Compute the flows θ and ϕ of V and W. Show that they do not commute, i.e. find explicit times s and t such that $\theta_s \circ \phi_t \neq \phi_t \circ \theta_s$.

Question 4 (Contraction). (4 points)

Let V be a k-dimensional vector space. For each $\omega \in \Lambda^p(V^*)$ we define $\iota_a(\omega) \in \Lambda^{p-1}(V^*)$ The *contraction* of ω with respect to $a \in V$) by

$$\iota_a\big(\omega\big)(v_1, \ldots, v_{p-1}) = \omega(a, v_1, \ldots, v_{p-1}).$$

For $p = 0$ we take $\iota_a(\omega) = 0$. Show that

$$\iota_a\big(\omega \wedge \eta\big) = \iota_a(\omega) \wedge \eta + (-1)^p \omega \wedge \iota_a(\eta)$$

for all p-forms ω and q-forms η on V.