Prof. Dr. Werner Müller
Dr. Fabian Meier

Wintersemester 2010/11

4. Übung Globale Analysis I

Abgabe am Montag, den 15. November, in der Vorlesungspause.

Bei Fehlern oder Fragen bitte eine eMail an: `brief`α`fabianmeier.de` .

Question 1 (integral curves on S^2). (3 points)

Show, that

$$X(x, y, z) = (-y, x, 0)$$

defines a smooth vector field on S^2. Determine the integral curves of X.

Question 2 (Vector fields on Submanifolds). (4 points)

Let M be a manifold and $N \subset M$ be an embedded submanifold. Furthermore, let $X \in C^\infty(TM)$ be a smooth vector field for which we have: $p \in N \Rightarrow X(p) \in T_pN$. Show:

1. $X\big|_N$ is a smooth vector field on N.
2. Let $\gamma\colon (-\varepsilon, \varepsilon) \to M$ be an integral curve of X with $\gamma(0) \in N$. Then we have $\operatorname{Im}\gamma \subset N$ and γ is an integral curve of $X\big|_N$.
3. Every smooth vector field Y on N is locally just the restriction of a smooth vector field X on M, i.e. for every point $p \in N$ there exists an open neighbourhood U of p in M and a vector field $X \in C^\infty(TU)$ with $Y\big|_{U\cap N} = X\big|_{U\cap N}$.

Question 3 (Properties of the Lie bracket). (4 points)

Prove Lemma 1.38!

Question 4 (Related Lie brackets). (5 points)

Let M, N be manifolds and $\phi\colon M \to N$ be a smooth map. Two vector fields $X \in C^\infty(M)$, $Y \in C^\infty(N)$ are called *ϕ-related*, if

$$Y(\phi(p)) = (d\phi)_p\big(X(p)\big).$$

For $i = 1, 2$ let $X_i \in C^\infty(M)$ and $Y_i \in C^\infty(N)$ be ϕ-related vector fields. Show, that $[X_1, X_2]$ and $[Y_1, Y_2]$ are also ϕ-related.