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Wintersemester 2010/11

11. Übung Globale Analysis I

Abgabe am Montag, den 17. Januar, in der Vorlesungspause.

Bei Fehlern oder Fragen bitte eine eMail an: `brief`α`fabianmeier.de` .

Question 1 (orientation — different definitions). (5 points)

Show that for a connected manifold M the following three conditions are equivalent:

1. M is orientable.
2. There exists an atlas $\mathcal{A} = \{(U_i, \phi_i) : i \in I\}$ of M so that for all $i, j \in I$ with $U_i \cap U_j \neq \emptyset$ we have:

$$\det\left(d(\phi_j \circ \phi_i^{-1})\big|_{\phi_i(x)}\right) > 0$$

   for $x \in U_i \cap U_j$.
3. There is an element $\omega \in \Lambda^n(M)$ so that $\omega(x) \neq 0$ for all $x \in M$.

Question 2 (Antipodal map). (3 points)

Show that the map $S^n \to S^n$, which is given by $x \mapsto -x$ (using the coordinates of $\mathbb{R}^{n+1}$) has degree $(-1)^{n+1}$.

Hint: Without proof you may use that the *standard volume form*

$$\sum_{j=1}^{n+1} (-1)^{j-1} x_j dx_1 \wedge \ldots \wedge dx_{j-1} \wedge dx_{j+1} \wedge \ldots \wedge dx_{n+1}$$

generates the nth cohomology of S^n.

Question 3 (normal vector field). (4 points)

Let M be an n-dimensional manifold for which we have an immersion into $\mathbb{R}^{n+1}$. Show that M is orientable if and only if there exists a non-vanishing normal vector field of M in $\mathbb{R}^{n+1}$.

Question 4 (The projective space). (4 points)

Prove that the real projective space $\mathbb{RP}^n$ is orientable if and only if n is odd.