Topology II — Course Description

Prof. Dr. Carl-Friedrich Bödigheimer summer term 2017

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1) **Content**

The course is an introduction into singular cohomology theory.
So it is already algebraic topology, although this word occurs as a course name only later in the master program; it is part of a longer series, namely

(0) Introduction into Geometry and Topology (summer term 2016)
(1) Topology I (winter term 2016/17)
(2) Topology II (summer term 2017)
(3) Algebraic Topology I (winter term 2017/18)
(4) Algebraic Topology II (summer term 2018)

We will define the singular cohomology groups $H^*(X)$ of a space X and prove that they constitute a cohomology theory and satisfy the axioms of Eilenberg and Steenrod.
Our next goal is to compute the cohomology groups of some important spaces like spheres, surfaces and projective spaces or some 3-manifolds.

Then we introduce homology groups $H_*(X;\mathbb{G})$ and cohomology groups $H^*(X;\mathbb{G})$ with coefficients in an arbitrary abelian group $\mathbb{G}$. This leads to the so-called Universal Coefficient Theorems relating $H^*(X;\mathbb{G})$ and $H_*(X;\mathbb{G})$ to $H_*(X;\mathbb{Z})$ and $H^*(X;\mathbb{Z})$.

The next goal is the homology and cohomology groups of products $X \times Y$. They can be expressed by the homology and cohomology groups of X and Y and this is called the Künneth Sequence. Then we introduce the homology and cohomology cross product ; and then the cup product in cohomology, which turns the sum of all cohomology groups into a graded ring. The cap product as an action of this ring on the sum of all homology groups.

Finally we study the homology and cohomology of manifolds, resulting in the famous Poincare duality.

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2) **Prerequisites**

The course assumes a good understanding of singular homology theory as taught in the course Topology I, including a basic knowledge of manifolds, and also a good knowledge of the fundamental group and covering spaces (see the book by Bredon and the book by Hatcher). We need the beginnings of group theory and modules over commutative rings, mainly principal ideal domains will be used (for both topics see chapters I - III in the book by Lang).

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3) **Recommended Literature**

There are many very good textbooks covering the topic. The books below cover singular homology and cohomology, thus the content of the course Topology I (winter term 2016/17) as well as the next course Topology II (summer term 2017).

- G. E. Bredon: *Topology and Geometry.* Springer Verlag (1993).
- A. Dold: *Lectures on Algebraic Topology.* Springer Verlag (1973).
- A. Hatcher: *Algebraic Topology.* Cambridge University Press (2002).
- S. Lang: *Algebra.* Addison-Wessley (1993^3).