Aufgaben zur Topologie

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Week 1 — Exercises and Revision to be done by: 25.10.2016

Exercise 0.1 (Homotopy)
Recall the definition of homotopy and relative homotopy. Homotopy classes.

Exercise 0.2 (Free and based homotopy of maps $\mathbb{S}^1 \to \mathbb{S}^1$)
Show that, for self-maps of $\mathbb{S}^1$, free and based homotopy are the same.

Exercise 0.3 (Group structure of the fundamental group)
Recall the definition and verify the group axioms of $\pi_1(X, x_0)$.

Exercise 0.4 (Dependence on the fundamental group)
(a) Formulate correctly the phrase: $\pi_1(X, x_0)$ depends only on the path component of x_0.
(b) Formulate correctly the phrase: If X is path-connected, then $\pi_1(X, x_0)$ is independent of x_0.

Exercise 0.5 (Degree)
Recall the definition and the basic properties of the function $\mathbf{grad} : \pi_1(\mathbb{S}^1, 1) \to \mathbb{Z}$.

Exercise 0.6 (Applications)
Recall the statements and proofs of
(a) the fundamental theorem of algebra,
(b) the Brouwer fixed point theorem (in dimension 2),
(c) the Borsuk-Ulam theorem,
(d) the hairy ball theorem.