Aufgaben zur Topologie I

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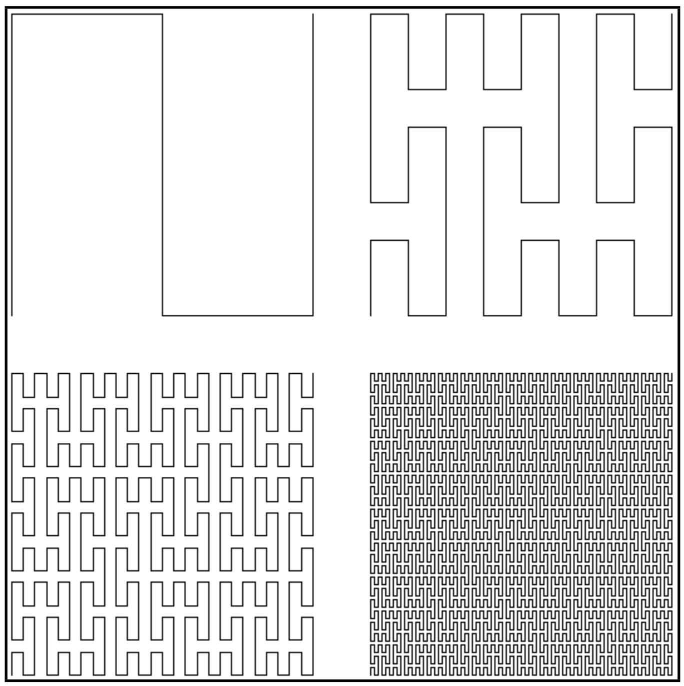
Wintersemester 2019/20

Blatt 10 due by: 18.12.2019



The first four curves of a sequence of curves, which converges to a Peano curve; this is a continuous and surjective curve from an interval onto a square. The first such curve was found by Guiseppe Peano (1858 - 1932). It made all naive ideas of dimension obsolete.

Exercise 10.1 (Products of spheres)
a) Assume $M = \mathbb{S}^{m_1} \times \mathbb{S}^{m_2}$ and $N = \mathbb{S}^{n_1} \times \mathbb{S}^{n_2}$. Show that if $M \cong N$, then $\{m_1, m_2\} = \{n_1, n_2\}$.
b) Assume $M = \mathbb{S}^{m_1} \times \ldots \times \mathbb{S}^{m_k}$, and $N = \mathbb{S}^{n_1} \times \ldots \times \mathbb{S}^{n_k}$. Show that if $M \cong N$, then $\{m_1, \ldots, m_k\} = \{n_1, \ldots, n_k\}$.

Exercise 10.2 (Complements of disks in $\mathbb{R}^n$)
Deduce from the Theorem on Complements of Disks in $\mathbb{S}^n$:
If $f\colon \mathbb{D}^r \to \mathbb{R}^n$ is an embedding, then $\widetilde{H}_k(\mathbb{R}^n - f(\mathbb{D}^r)) = \mathbb{Z}$ for $k = n-1$ and $\widetilde{H}_k(\mathbb{R}^n - f(\mathbb{D}^r)) = 0$ else.

Exercise 10.3 (Union of disks and their complement in $\mathbb{S}^n$)
Let $K = A \cup B$ be a subset of $\mathbb{S}^n$ and assume that A, B and the intersection $A \cap B$ are homeomorphic to disks. Show that $\mathbb{S}^n - K$ is acyclic.

Exercise 10.4 (General Jordan Curve Theorem in $\mathbb{R}^n$.)
Let $f\colon \mathbb{S}^r \to \mathbb{R}^n$ be an embedding. What can we say about the homology of its complement $\mathbb{R}^n - f(\mathbb{S}^r)$?

Camille Jordan (1838 - 1922), french mathematician, not only famous for the Jordan normal form of matrices.

Exercise 10.5 (Complements of two spheres)
Let $A, B \subset \mathbb{S}^n$ be disjoint subsets and assume A is an embedded p-sphere and B an embedded q-sphere. Compute the homology of the complement $\mathbb{S}^n - (A \cup B)$.

Exercise 10.6* (Acyclic complement property)
For any space X we consider a compact space K and all its possible embeddings $f\colon K \to X$. If these complements $X - f(K)$ are always acyclic, we say K *has the acyclic complement property for* X, and write ACP for short.
Are the following statements true ?

a) If K has the ACP for X, then also for any $X' \cong X$.

b) If K has the ACP for X, then also any $K' \cong K$.

c) If $K = K_1 \cup K_2$ for two compact subspaces K_1 and K_2, and all of K_1, K_2 and $K_1 \cap K_2$ have the ACP for X, then K also has the ACP for X.

d) If K_1, K_2 and $K = K_1 \cup K_2$ have the ACP for X, then also $K_1 \cap K_2$.

e) If $K_1 \supset K_2 \supset \ldots$ is a descending chain of spaces, all having the ACP for X, then also $K = \bigcap K_i$.