Topology I + II

Course Description

Prof. Dr. Carl-Friedrich Bödigheimer — winter term 2019/20 + summer term 2020

---

0) **The main series of lecture courses in topology**

The lecture course *Topology I* is an introduction into singular homology. It will be followed by *Topology II* in the summer term 2020, which is an introduction into singular cohomology theory.
Both lecture courses are part of a long series of altogether four lecture courses, or even five, if we regard the lecture course *Introduction into Geometry and Topology* as the beginning:

(0) *Introduction into Geometry and Topology*	(summer term 2019, Prof. Hamenstädt)
(1) *Topology I*	(winter term 2019/20, Prof. Bödigheimer)
(2) *Topology II*	(summer term 2020, Prof. Bödigheimer)
(3) *Algebraic Topology I*	(winter term 2020/21, Prof. Bödigheimer)
(4) *Algebraic Topology II*	(summer term 2021, Prof. Bödigheimer)

The courses (1) and (2) are already algebraic topology, although this word does not occur in their course name, but in the course name of the much more advanced courses (3) and (4). Theses latter lecture courses belong to our Master Program. Lecture courses (3) and (4) will treat homotopy theory and generalized homology and cohomology theories and several more advanced topics.

Content

In *Topology I* we will define the singular homology groups $H_*(X)$ of a space X and prove that they constitute a homology theory. And in *Topology II* we will define the singular cohomology theory H^* with the product structure; we will investigate its relation to H_* and prove at the end the Poincare duality for manifolds.

The first chapter treats general chain complexes over the ring of integers and their homology. This will then be applied to the singular chain complex of a space. We need to show that this is a homology theory by proving the Eilenberg-Steenrod axioms (functoriality, homotopy invariance, excision or Mayer-Vietoris property, and the long exact sequence of a pair of spaces).

In order to compute homology groups we look at spaces with a decomposition into simplices or into cells (polyhedra resp. CW complexes). With this we can compute the homology groups of many important spaces, like spheres, surfaces, 3-manifolds, projective spaces, lens spaces and many more.

We will introduce homology groups $H_*(X; \mathbb{G})$ with coefficients in an abelian groupp $\mathbb{G}$ and study their relation to homology groups with coefficients $H_*(X; \mathbb{Z})$, the so-called universal coefficient theorems.

The next goal is the homology groups of products $X \times Y$. They can be expressed by the homology and cohomology groups of X and Y and this is called the Künneth Sequence.

We will also consider the relation between homology groups and homotopy groups.

---

2) **Prerequisites**

The course assumes a good understanding of point set topology as taught in the course *Introduction to Geometry and Topology*, including a basic knowledge of manifolds, and also a good knowledge of the fundamental group and covering spaces. We also need the basics of group theory and of modules over commutative rings, mainly principal ideal domains.

---

3) **Recommended Literature**

There are several very good textbooks for the topic. The books below cover singular homology and cohomology, thus the content of the lecture course Topology I as well as the next course Topology II.

- G. E. Bredon: *Topology and Geometry.* Springer Verlag (1993).
- A. Dold: *Lectures on Algebraic Topology.* Springer Verlag (1973).
- A. Hatcher: *Algebraic Topology.* Cambridge University Press (2002).