Winter term 2018-19
V5B2 - Selected topics in Analysis and PDE
Geometric Optimal Control

Do 10(c.t.)–12 N 0.008 - Neubau

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It is planned to trace the connections of Optimal Control Theory with other branches of Analysis and Applied Mathematics. The main aims are:

- Examples of control systems with applications in Engineering and Math Physics.
- Pontryagin Maximum Principle (PMP) and bang-bang controls.
- Hamilton-Jacobi-Bellman (HJB) equations and value functions.
- Resonances and their optimization in the Pareto sense (following [AK, KLV])
- The minimum-time control approach to resonance optimization (following [KKV]).

If there is time, the following advanced topics will be discussed: proximal solution to HJB-equations on manifolds (following [CV]), the proof of PMP, stochastic optimal control (following [E]).

Prerequisites: Analysis I–II, basic knowledge of ODE (existence and uniqueness, systems of ODE, see [Braun, Sections 1.9 and 4.1-5]).

Basic knowledge of the following topics is useful, but not necessary: Banach spaces of continuous/Lebesgue integrable functions, weak-compactness, smooth manifolds (main definitions).

Literature available in Internet

[AK] Albeverio S., Karabash I.M., On the multilevel internal structure of the asymptotic distribution of resonances, preprint arXiv:1807.02889, https://arxiv.org/pdf/1807.02889, (gives some idea about sets of 1-D and 3-D resonances)

[KLV] I.M. Karabash, O.M. Logachova, I.V. Verbytskyi, Nonlinear bang-bang eigenproblems and optimization of resonances in layered cavities. Integr. Equ. Oper. Theory 88(1) (2017), 15–44; see also the preprint https://arxiv.org/pdf/1508.04706, (gives some idea about resonances in optical cavities and their optimization).

[KKV] I.M. Karabash, H. Koch, I.V. Verbytskyi, Pareto optimization of resonances and minimum-time control, preprint arXiv:1808.09186, https://arxiv.org/pdf/1808.09186, (shows how to reduce resonance optimization to optimal control)

[CV] I. Chryssochoos, R.B. Vinter, Optimal control problems on manifolds: a dynamic programming approach. JMAA 287(1), (2003), 118–140, (a simple way into HJB equations on manifolds)

[E] L.C. Evans, Lecture notes of the course ”An Introduction to Mathematical Optimal Control Theory”, https://math.berkeley.edu/ evans/control.course.pdf, (a short course for undergraduates with advanced applications)

[B] J.F. Bonnans, Course on Optimal Control “Part I: the Pontryagin approach”, http://www.cmap.polytechnique.fr/ bonnans/notes/oc/ocbook.pdf, (an advanced course aimed on PMP and numerics)

Literature available in the math library

[AS] A.A. Agrachev, Y. Sachkov, Control theory from the geometric viewpoint. Springer, 2013 (concise and on manifolds)

[BC] M. Bardi, I. Capuzzo-Dolcetta, Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations. Springer, 2008 (detailed and centered around HJB eq-s)

[CS] Cannarsa, P., Sinestrari, C., Semiconcave functions, Hamilton-Jacobi equations, and optimal control. Springer, 2004 (the 2 chapters on optimal control is a short and accessible introduction).

[SL] H. Schättler, U. Ledzewicz, Geometric optimal control: theory, methods and examples. Springer, 2012 (many examples, detailed extremal synthesis, PMP on manifolds)

[LM] Lee, E.B. and Markus, L., Foundations of optimal control theory. Wiley, 1967 (one of the main textbooks before 2000).

[Braun] M. Braun, Differential equations and their applications, Springer, 1975 (a textbook on ODE available in abundance in the math library)