**Mathematical Institute, Bonn**
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## Real and Harmonic Analysis

Exercise Sheet 11 | 14 Jan, 2015 | Turn in by: 10:00 21 Jan, 2015

### Problem 30 (10 points)

Let $0 < p \leq 1$ and let $a$ be a $p$ atom. Prove that $a \in \mathcal{H}^p$.

### Problem 31 (10 points)

Let $K(x, y) = g(x - y)$ be the convolution kernel of the operator $T : L^2 \to L^2$ which satisfies

$$|g(z)| \leq c|z|^{-n}, |Dg(z)| \leq c|z|^{-n-1}$$

Prove that $T$ defines a bounded operator on $\mathcal{H}^1$. Hint: First study the convolution kernel of $\varphi_t * T$. Then verify the assertion for atoms.

### Problem 32 (10 points)

Let $0 < p < 1$. Find an element of $\mathcal{H}^p$ which is supported on the union of two different points. Hint: First try the difference of Dirac measures for $p$ close to $1$.

**All answers should be fully justified. The solution to a given point may depend on the solutions of the previous points: if you didn't manage to prove a required statement you can still use it in the subsequent points.**
**Working in small groups is encouraged (no more than 4 people/group). You may hand in only one answer sheet per group. Put all the names on the sheet. During the exercise sessions students must know how to solve all the exercises done by their group, otherwise the assignment is void.**