University of Bonn
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Due on Friday 22 May. Submit your solutions in groups of two or three. Please use the correct subject line *real_and_harmonic_2020* in your email, as instructed in the first exercise sheet. Submit the tex-file.

Problem 1. Let $F : \mathcal{D} \to \mathbb{C}$ satisfy the martingale identity.

a) Prove that if $\sup_I |F(I)| < \infty$, then F is the martingale extension of an absolutely continuous Radon measure.

b) Prove that if $\sum_I |I||\Delta F(I)|^2 < \infty$, then F is the martingale extension of an absolutely continuous Radon measure.

Problem 2. Let m be a Radon measure on $[0, 1)$ and F its martingale extension. Show that

$$\sum_{k\in\mathbb{Z}} |\widehat{m}(k)|^2 = \sum_{I\in\mathcal{D}} |I||\Delta F(I)|^2$$

in the sense that if one of the two sides is finite, then so is the other and both sides are equal.