Summer term 2020 | University of Bonn
Real and harmonic analysis | Prof. Dr. Christoph Thiele
Exercise sheet 2 | Dr. Olli Saari

Due on Friday 1 May 2020. Hand in in groups of two or three (not alone, you are too many). We will discuss how to form groups in the exercise session. You can still work alone if you wish, but do the write-up collaboratively.

Problem 1. Let $\lambda : [0, 1) \to \mathbb{R}$ be bounded and increasing. Define

$$\lambda_l(x) = \lim_{y \to x-} \lambda(y), \quad x \neq 0, \qquad \lambda_r(x) = \lim_{y \to x+} \lambda(y), \quad x \neq 1$$

and

$$\lambda_l(0) = \lim_{y \to 1-} \lambda(y), \qquad \lambda_r(1) = \lim_{y \to 0+} \lambda(y).$$

Identify λ with a function on the unit circle, and let F be its harmonic extension to the unit disc. Show that

$$\lim_{r \to 1-} F(r, \theta) = \frac{1}{2}(\lambda_l(\theta) + \lambda_r(\theta)).$$

Problem 2. Consider a real Banach space V (real vector space that is normed and complete). Suppose that the parallelogram law holds: For all $x, y \in V$

$$2(\|x\|^2 + \|y\|^2) = \|x + y\|^2 + \|x - y\|^2.$$

Consider the form on $V \times V$ defined through

$$(x, y) \mapsto \langle x, y \rangle := \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2).$$

Show that $\langle \cdot, \cdot \rangle$ is an inner product (linear, positive definite, symmetric).
Hint: start by showing $\langle x + y, z \rangle + \langle x - y, z \rangle = 2\langle x, z \rangle$