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Partial Differential Equations and Modelling

Sheet Nr.4 Due: 19.05.2017

Exercise 1

Let $\mu = (2\pi)^{-\frac{1}{2}} e^{-x^2/2} dx$. Prove that the orthogonal polynomials are the Hermite polynomials. Determine the a_n and b_n.

Exercise 2

Consider the linear Schrödinger equation

$$i\partial_t u + \Delta u = 0, \qquad u(t=0,x) = \phi(x), \quad x \in \mathbb{R}^d.$$

Let $\phi \in \mathcal{S}(\mathbb{R}^d)$. Find a solution as a convolution with ϕ by first applying a Fourier transform in x.

Exercise 3

Find examples of operators

a) $T \in L(H)$, so that $\langle Tx, x\rangle \in [0,\infty)$ for all $x \in H$ but T is not normal.

b) Let $K \subset \mathbb{C}$ be compact. Find T so that K is its spectrum.

c) Let (λ_j) a summable sequence of positive numbers and (z_j) a bounded sequence of pairwise disjoint complex numbers. Find T and a cyclic vector so that $\mu = \sum\limits_{j=1}^{\infty} \lambda_j \delta_{z_j}$.

Exercise 4

Prove the following statements.

a) Characterize the $n \times n$ matrices for which there exists a cyclic vector.

b) Let T be selfadjoint. Prove that there exist T_n selfadjoint, $T_n \to T$ in $L(H)$ so that there exists a cyclic vector ϕ_n of T_n and such that the measure μ_n is a sum of Dirac measures.