ARithmetische Geometrie OberSeminar
Analytic de Rham stacks

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The goal of this ARGOS is to study the works [RC24a] and [ABLB+25] on analytic de Rham stacks.

1 Introduction

The de Rham stack for algebraic varieties over a field K of characteristic 0 was introduced by Simpson in [Sim96]. Given an algebraic variety X, its de Rham stack $X^{\mathrm{dR,alg}}$ is described as the functor $X^{\mathrm{dR,alg}}\colon \mathsf{Alg}_K \to \mathsf{Sets}$ sending a K-algebra A to $X(A^{\mathrm{red}})$ where $A^{\mathrm{red}} = A/\mathrm{Nil}(A)$ is the reduction of A. The formation of $X \mapsto X^{\mathrm{dR,alg}}$ is a geometrization of the de Rham cohomology in the sense that the coherent cohomology of $X^{\mathrm{dR,alg}}$ is naturally equivalent to the sheaf cohomology of the de Rham complex, more generally, the category $\mathrm{D}(X^{\mathrm{dR,alg}})$ of quasi-coherent sheaves on the algebraic de Rham stack of X is naturally equivalent to the category of algebraic D-modules on X. The de Rham stack of algebraic varieties, and its natural extension to other geometric gadgets such as algebraic stacks, has been a fundamental object in geometric representation theory, eg. for a reformulation of the Beilinson-Bernstein localization [BB81], and for the geometric Langlands program [GR14, GR24].

In recent years, an analytic analogue of Simpson's de Rham stack has been introduced in analytic geometry, the so called analytic de Rham stack [RC24a], [Sch24b]. In p-adic geometry, this object geometrizes the theory of D-cap modules of Ardakov and Wadsley [AW19], while in the complex or real case the analytic de Rham stack agrees with the Betti stack whose theory of quasi-coherent sheaves is just complex or real valued sheaves. The analytic de Rham stack has a similar modular definition as its algebraic counterpart; given X a rigid variety over $\mathbb{Q}_p$ its analytic de Rham stack X^{dR} is the (sheafification for the !-topology of the) functor $X^{\mathrm{dR}}\colon \mathsf{Ring}^b_{\mathbb{Q}_p} \to \mathsf{Set}$ from the category of *bounded $\mathbb{Q}_p$-algebras* to sets mapping A to $X(A^{\dagger-\mathrm{red}})$, where $A^{\dagger-\mathrm{red}} = A/\mathrm{Nil}^{\dagger}(A)$ is the *dagger reduction* of A, and $\mathrm{Nil}^{\dagger}(A)$ is the *dagger nil-radical of* A. Here we use the notion of bounded $\mathbb{Q}_p$-algebras from [RC24a]; this notion captures the idea that all elements of A have uniformly bounded norm. Heuristically, $\mathrm{Nil}^{\dagger}(A)$ are the elements $a \in A$ of *spectral norm* 0.

The theory of the analytic de Rham stack satisfies very strong descent properties – namely, arc-descent – and hence extends from rigid spaces to perfectoid spaces and even arc-stacks. This allows one to make sense of de Rham stacks of diamonds, Fargues-Fontaine curves, Div^1, Bun_G, etc. This level of generality provides a realization of the theory of Berkovich motives [Sch24a], which can then be applied to construct a concrete stacky realization of the geometrization of the motivic local Langlands correspondence of [Sch25] with (locally constant) $\mathbb{Q}_p$-coefficients. With this motivation in mind, the work [ABLB$^+$25] extends the theory of analytic de Rham stacks to the so called *qfd arc-stacks*, and studies in detail the de Rham stack of (the Fargues-Fontaine stack of) Div^1, relating its category of quasi-coherent sheaves with the so called *p-adic differential equations* or (φ, ∂)-modules over the Robba ring, eg. as in [Ked04].

More precisely, given X an arc-stack over $\mathbb{F}_p$, consider its relative Fargues-Fontaine curve $\mathcal{Y}_X := X \times_{\mathrm{Spd}(\mathbb{F}_p)} \mathrm{Spd}(\mathbb{Q}_p)$ and $\mathrm{FF}_X = \mathcal{Y}_X/\varphi_X^{\mathbb{Z}}$. Then $\mathrm{FF}_X \to \mathrm{Spd}(\mathbb{Q}_p)$ is an arc-stack over $\mathbb{Q}_p$. We define the *Hyodo-Kato stack of* X, denoted as X^{HK}, to be the de Rham stack

$$X^{\mathrm{HK}} := (\mathrm{FF}_X)^{\mathrm{dR}}$$

over $\mathbb{Q}_p$. In the case of $X = \mathrm{Spd}(\mathbb{Q}_p)$, one has that $\mathrm{Div}^1 = X/\varphi^Z$, and $(\mathrm{Spd}(\mathbb{Q}_p))^{\mathrm{HK}} = (\mathrm{Div}^1 \times_{\mathrm{Spd}(\mathbb{F}_p)} \mathrm{Spd}(\mathbb{Q}_p))^{\mathrm{dR}}$ is what we meant by the de Rham stack of Div^1. The cohomological properties of the stacks X^{HK} are also studied in [ABLB$^+$25], establishing for example Poincaré duality. The name of the stacks X^{HK} come from the fact that they realize geometrically Hyodo-Kato and rigid cohomology, eg. as in [BGV24] or [CN25]; the precise formulation of this fact will be studied in future works.

The definition of de Rham stacks for general arc-stacks involves a new framework in p-adic geometry called *Gelfand Stacks*. The category of Gelfand Stacks is a more manageable version of the category of analytic stacks, and is constructed from a full subcategory of bounded rings, called Gelfand rings, which are heuristically bounded rings whose completions are Banach algebras. For technical reasons – and because this is sufficient for our applications – we also restrict to Berkovich geometry rather than Huber's adic geometry. This replaces a rigid space X with its Huber's compactification $\overline{X}$, so that the analytic de Rham stack X^{dR} coincides with the de Rham stack $(X^\dagger)^{\mathrm{dR}}$ of any *overconvergent structure* $X^\dagger$ *on* X. This is actually desirable, as only the latter has reasonable cohomology. Moreover, the choice of working in Berkovich geometry instead of adic geometry is harmless for most of the applications of the theory; for instance, all the stacks appearing in the geometrization of the Langlands program are partially proper.

Prerequisites: Familiarity with condensed mathematics is assumed, eg. as in [CS19]. Knowledge of analytic stacks is useful but not necessary; there will be preliminary talks on this topic. Many main ideas and definitions of the seminar can be explained and stated without an explicit reference to higher category theory, nonetheless, the foundations and techniques of the subject rely heavily on it. The participant is also expected to have some familiarity with the arc, v and proétale topologies on Banach and perfectoid rings [Sch22, Sch24a].

Other useful references: [And21], [Man22], [HM24], [Sch23], [CS22], [CS20], [CS19], [RC24b].

2 Distribution of the talks

Talk 1 (October 23, 12:15–14, Room 0.006, Endenicher Allee 60). Introductory talk.

Overview talk and distributions of talks. Please attend this meeting if you want to give a talk.

Talk 2. Recollections on Analytic Stacks

Introductory talk on the theory of analytic stacks. Recall the definition of the category of analytic rings, the quasi-coherent six functor formalism on analytic rings, the !-topology and the category of analytic stacks. Reference for these are [CS24], [RC24b, Section 6] or [ABLB+25, Section 4.2]. Recall how algebraic stacks can be seen as analytic stacks ([RC24b, Section 6.4] and [ABLB+25, Lemma 3.2.1]), deduce the Betti realization of condensed anima as analytic stacks ([ABLB+25, Section 3.2], [RC24b, Section 6.5] or [Sch24b, Chapter II.1]) and describe its category of quasi-coherent sheaves (eg. after [HM24]). Finally, recall how adic spaces can be seen as analytic stacks ([And21]).

Talk 3. Recollections in Serre and Cartier duality

This is a continuation of the previous talk in a more example-based presentation. Briefly recall the definition of suave and prim maps in six functor formalisms (but do not expend much time on that). Describe the six functors for the map $f\colon \mathrm{AnSpec}(\mathbb{Z}[T]_{\square}) \to \mathrm{AnSpec}(\mathbb{Z}_{\square})$ and prove that $\mathrm{AnSpec}(\mathbb{Z}[T]_{\square})$ is cohomologically smooth ([CS19, Lecture VIII] and [RC24b, Proposition 7.1.11]). After base change to $\mathbb{Z}_p$, deduce that $\mathrm{Spa}(\mathbb{Q}_p\langle T\rangle, \mathbb{Z}_p\langle T\rangle) \to \mathrm{Spa}(\mathbb{Q}_p)$ is cohomologically smooth for the solid six functor formalism.

Then, following [RC24a, Section 3.6] show that morphisms of smooth rigid spaces are cohomologically smooth for the solid six functor formalism (reduce to local complete intersections and affinoid discs). Finally, sketch the proof of the Cartier duality between the algebraic affine line $\mathbb{G}_{a,\mathbb{Q}}^{\mathrm{alg}} = \mathrm{Spec}(\mathbb{Q}[T])$ and its formal completion at zero $\widehat{\mathbb{G}}_{a,\mathbb{Q}}$, and between the analytic affine line $\mathbb{G}_{a,\mathbb{Q}_p}^{\mathrm{an}}$ and its overconvergent neighbourhood at zero $\mathbb{G}_{a,\mathbb{Q}_p}^{\dagger} = \mathrm{AnSpec}(\mathbb{Q}_p\langle T\rangle_{\leq 0})$ where $\mathbb{Q}_p\langle T\rangle_{\leq 0} = \varinjlim_n \mathbb{Q}_p\langle \frac{T}{p^n}\rangle$ (eg. [Bha22, Proposition 2.2.13] and [RC24a, Section 4]). Using this, prove that $B\widehat{\mathbb{G}}_{a,\mathbb{Q}} \to \mathrm{Spec}(\mathbb{Q})$ is suave and prim, same for $B\mathbb{G}_{a,\mathbb{Q}_p}^{\dagger} \to \mathrm{Spa}(\mathbb{Q}_p)$.

Talk 4. Gelfand rings

Introduce the category of bounded rings over $\mathbb{Q}_p$ as in [RC24a] and [ABLB+25, Section 2]; focus on solid rings with the induced analytic ring structure. Given a bounded ring A, define the solid subspaces $A^{\leq r}$ of elements of norm $\leq r$ in A, as well as the dagger-nilradical $\mathrm{Nil}^{\dagger}(A)$, the dagger-reduction $A^{\dagger-\mathrm{red}}$ and the uniform completion A^u. State in particular [ABLB+25, Proposition 2.2.20, 2.3.6 and Corollary 2.3.7].

Then, following [ABLB+25, Section 3], introduce the category of Gelfand rings, define the Berkovich spectrum of a Gelfand $\mathbb{Q}_p$-algebra and construct the map $\mathrm{GSpec}(A) \to \mathcal{M}(A)_{\mathrm{Betti}}$ in the finite dimensional case. More precisely, the speaker should discuss [ABLB+25, Propositions 3.1.9, 3.1.16, 3.1.19 and 3.2.10]. We suggest the speaker to keep the discussion non-technical, only providing the key steps in the proofs.

Finally, state (but not prove) the Fredholm property of Gelfand rings of [ABLB+25, Proposition 3.3.5]. Here and elsewhere, restrict to static rings whenever this simplifies the discussion.

Talk 5. Arc- and Gelfand Stacks

The goal of this talk is to construct the categories of arc and Gelfand stacks, and the perfectoidization functor $(-)^{\diamond}\colon \mathrm{GelfStk} \to \mathrm{ArcStk}_{\mathbb{Q}_p}$. More specifically, following [ABLB+25, Section 4.1] introduce the category of (light) *arc-stacks*. Give the example of the realization of condensed anima in arc-stacks of [ABLB+25, Example 4.1.9].

Define the category of Gelfand stacks following [ABLB+25, Section 4.2], and construct the perfectoidization functor of [ABLB+25, Lemma 4.5.1] as a left adjoint of a morphism of topoi. Prove that $(-)^{\diamondsuit}$ is itself a right adjoint of a functor $\widehat{(-)}\colon \mathsf{ArcStk}_{\mathbb{Q}_p} \to \mathsf{GelfStk}$ sending an affinoid perfectoid $\mathcal{M}_{\mathrm{arc}}(A)$ to $\mathrm{GSpec}(A)$ ([ABLB+25, Proposition 4.5.5]).

Finally, construct the *Fargues-Fontaine stack functor* $\mathcal{Y}\colon \mathsf{ArcStk}_{\mathbb{F}_p} \to \mathsf{GelfStk}$ ([ABLB+25, Corollary 4.4.10]). Mention the main ingredients for its hyperdescent, and relate with the work of [AM24, ALBM24].

In this talk the speaker is invited to black-box the technical descendability results of perfectoid and Gelfand rings on Sections 4.4-4.6 of [ABLB+25].

Talk 6. Analytic de Rham stacks I: derived Berkovich spaces

Introduce the category of *derived Berkovich spaces* as in [ABLB+25, Section 4.3], proving in particular that derived Berkovich spaces embed fully faithfully in Gelfand stacks [ABLB+25, Proposition 4.3.3]. Define rigid-étale and smooth maps of derived Berkovich spaces [ABLB+25, Definition 4.3.4], and the notion of †-rigid space [ABLB+25, Definition 4.3.6].

Then define the *big de Rham stack* $(-)^{\mathrm{DR}}$ as a right adjoint of the perfectoidization functor [ABLB+25, Definition 4.5.3]. Prove that $(-)^{\mathrm{DR}}$ is a fully faithful embedding of arc-stacks into Gelfand stacks. Proceed to study the big de Rham stack of derived Berkovich spaces in [ABLB+25, Section 4.7]. More precisely, introduce the notion of †-formally étale and smooth of [ABLB+25, Definition 4.7.2], give the examples [ABLB+25, Example 4.7.6], and give a sketch of the proofs of [ABLB+25, Propositions 4.7.11 and 4.7.13] describing the basic structure of the de Rham stack of a derived Berkovich space. Finally, finish with a detailed explanation of the example of the de Rham stack of the perfectoid multiplicative group of [ABLB+25, Example 4.7.16].

Similar as in previous talks, the speaker is invited to assume the necessary descendability results of Sections 4.4-4.6 of [ABLB+25].

Talk 7. Analytic de Rham stacks II: arc-descent

We continue with the study of the analytic de Rham stack. Discuss arc-descent of the de Rham stack. Recall the construction of the functor $\underline{(-)}\colon \mathsf{CondAni} \to \mathsf{ArcStk}_{\mathbb{Q}_p}$ of [ABLB+25, Example 4.1.9]. Introduce the notion of quasi-finite dimensional morphism of arc-stacks [ABLB+25, Definition 4.1.11], and that of Gelfand rings [ABLB+25, Definition 4.2.14]. From this, redefine the perfectoidization $(-)^{\diamond}$ and de Rham stack $(-)^{\mathrm{dR}}$ for *qfd arc- and Gelfand stacks* (here only a definition will suffice).

In the rest of the talk, justify the set up of qfd stacks in order to prove arc-descent of the de Rham stack. For this, introduce the categories of nilperfectoid and (strictly) totally disconnected Gelfand rings of [ABLB+25, Definition 4.6.1]. The speaker should mention (without proof) the descendable properties of Gelfand rings of [ABLB+25, Proposition 4.6.4], in particular state that a qfd Gelfand ring admits a !-cover by a strictly totally disconnected nilperfectoid ring. Mention [ABLB+25, Example 4.5.9], the counter example of a compact Hausdorff space X such that $X_{\mathrm{Betti}} \to (\underline{X})^{\mathrm{DR}}$ is not an isomorphism, and hence that the *big de Rham stack* does not satisfy arc-descent. On the other hand, give a sketch of the arc-descent of the de Rham stack in the qfd setting of [ABLB+25, Theorem 4.8.6]. Finish by mentioning that the de Rham stack actually satisfies arc-hyperdescent in the qfd setting ([ABLB+25, Section 5.6]), but do not give a proof.

Talk 8. Six functors on analytic de Rham stacks I: proper and smooth maps

After having discussed some generalities on the analytic de Rham stack we will study more carefully its theory of six functors. From now on we shall assume that all the arc and Gelfand stacks are qfd, in particular, the de Rham stack satisfies arc-descent.

Define morphisms *locally of quasi-finite dimension* (lqfd) [ABLB+25, Definition 5.1.3], and prove [ABLB+25, Proposition 5.1.4]. Deduce that all morphisms between rigid spaces are !-able for the de Rham stack. Define *étale* and *smooth* morphisms of arc-stacks as in [ABLB+25, Definition 5.1.6], and prove that étale (resp. smooth) morphisms of arc-stacks are cohomologically étale (resp. cohomologically smooth). Explain the theory of first Chern classes of [ABLB+25, Remark 5.1.10] and use [Zav21] to prove Poincaré duality for the six functor formalism of analytic D-modules for rigid spaces.

Talk 9. Six functors on analytic de Rham stacks II: more properties

In this talk we will prove the comparison between the cohomology of the analytic de Rham stack and the sheaf cohomology of the de Rham complex in [ABLB+25, Section 5.2]. The speaker is welcome to present the comparison of [ABLB+25, Proposition 5.2.1] in detail, and give a sketch of the comparison via the filtered de Rham stack of [ABLB+25, Remark 5.2.3].

Next, prove that the de Rham stack of the perfectoid unit disc is cohomologically smooth following [ABLB+25, Section 5.7], using as a black box the technical lemma in abstract six functors of Section 5.3. Finally, explain [ABLB+25, Proposition 5.4.10] concerning vector bundles on de Rham stacks on derived Berkovich spaces, using [ABLB+25, Proposition 5.4.9] as a black box.

Talk 10: de Rham stacks of Fargues-Fontaine curves

Introduce the Hyodo-Kato stacks of [ABLB+25, Section 6]. Prove [ABLB+25, Proposition 6.1.12] (focus on the case $X = \mathbb{Q}_p$ and W any algebraic field extension of $\mathbb{Q}_p$). Then, present [ABLB+25, Lemma 6.2.1] describing different examples of Hyodo-Kato stacks. Also discuss the computations of [ABLB+25, Lemmas 6.2.2, 6.2.3, 6.2.5 and 6.2.9] which provide several examples of computations in Hyodo-Kato stacks, particularly the construction of a theory of Chern classes for Hyodo-Kato cohomology. Finally, state (but not prove) Poincaré duality for the Hyodo-Kato cohomology [ABLB+25, Theorem 6.3.1].

Talk 11. The p-adic monodromy theorem I: construction

First, give the statement of the p-adic monodromy theorem, and its interpretation using de Rham stacks of Fargues-Fontaine curves. Start by defining Robba rings following [ABLB+25, Section 7.2]. Then state the classical p-adic monodromy [Ked04, Theorem 1.1] (also known as Crew's conjecture). Next, follow [ABLB+25, Section 7.1] and sketch the construction of the main stacks involved in the geometric statement of the p-adic monodromy [ABLB+25, Theorem 7.1.1]. Explain the proof strategy of [ABLB+25, Remark 7.1.4]. Coming back to [ABLB+25, Section 7.2], state (and not prove) [ABLB+25, Proposition 7.2.8] relating φ-modules over the Robba ring and vector bundles on $\mathbb{Q}_p^{\mathrm{cyc},\mathrm{HK}}$, then deduce that the original formulation of Crew's conjecture is implied by the geometric formulation of [ABLB+25, Theorem 7.1.1]. Finally, discuss in detail the construction of the p-adic monodromy map of [ABLB+25, Construction 7.3.6] including all the previous discussion in Section 7.3 (but not [ABLB+25, Lemma 7.3.7]).

Talk 12. The p-adic monodromy theorem II: proof of the theorem

Finally, we give a proof of the p-adic monodromy theorem. First, for completeness of the previous talk, prove [ABLB+25, Lemma 7.3.7] computing the de Rham cohomology of the Tate curve; this produces another way to construct the extension class defining the p-adic monodromy map. Then prove fully faithfulness following [ABLB+25, Section 7.4] (here the speaker is welcome to skip proofs and just mention the technical reductions of [ABLB+25, Lemmas 7.4.2, 7.4.3 and 7.4.4]). Explain in detail the proof of [ABLB+25, Proposition 7.4.6]; assume the technical computation of Lemma 7.4.7, but mention that it is the only place where one needs to solve an explicit differential equation. Next, explain the proof of Tsuzuki's theorem in detail, this amounts to prove [ABLB+25, Proposition 7.5.1] over $\breve{\mathbb{Q}}_p$, and then deduce the p-adic monodromy theorem of [ABLB+25, Theorem 7.5.3].

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